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ROYAL ASTRONOMICAL SOCIETY.

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No. 3

H. F. NEWALL, Esq., M.A., F.R.S., PRESIDENT, in the Chair.

Henry Boase Austin, J.P., Government Buildings, Bloemfontein,
Orange River Colony, South Africa ;

W. Geoffrey Duffield, Physical Laboratory, Manchester ;

Wm. Earnshaw Etzel, B.S., Litt.L., M.S.M.F., Professor of
Sciences, St Bernard's, Rochester, N.Y., U.S.A. ;

John M. Field, 1 Hart Street, Edinburgh ;

James D. Maddrill, Ph.D., International Latitude Observatory,
Ukiah, California, U.S.A. ;

William Henry Rees, B.Sc., County School, Pontypridd, and
2 Craigwen Place, Pontypridd, South Wales ;

Thomas James Forrester Smith, Newstead, Wavertree,
Liverpool ; and

Captain James Weir, F.R.G.S., Examiner of Masters and
Mates in Navigation and Seamanship to the Board of Trade,
5 Clive Terrace, Penarth, South Wales,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows
of the Society, the names of the proposers from personal knowledge
being appended :—

Frederic Hermann Albert Alfred Buss, 2 Lansdowne Terrace,
Grosvenor Square, Ashton-on-Mersey, near Manchester
(proposed by A. Fowler) ;

Arthur du Pré Denning, M.Sc., Ph.D., Lecturer in Physics,
Birmingham University, 18 Lightwoods Hill, Birmingham
(proposed by William Briggs) ;

Herbert Shaw, Royal College of Science, South Kensington, S.W.
(proposed by A. Fowler); and
James Henry Worthington, Student in the University of Oxford,
Bindon, Wellington, Somerset (proposed by H. H. Turner).

Sixty presents were announced as having been received since the last meeting, including, amongst others:—

E. T. Whittaker, *The Theory of Optical Instruments*, presented by the author; framed photograph of 14th century clock restored by J. J. Hall, presented by Mr. Hall; *Calendrier Perpétuel*, presented by M. Bosson.

Seven transparencies of Sun-spots, Comet, Planets, and Nebulæ, from negatives taken at the Royal Observatory, Greenwich, presented by the Astronomer Royal.

J. C. Kapteyn, *On the number of stars of determined magnitude and determined galactic latitude* (Pub. Astron. Laboratory, Groningen, No. 18), presented by Professor Kapteyn.

On the Lunar Inequalities due to Planetary Action.

By Ernest W. Brown, D.Sc., F.R.S.

1. I have lately finished the computation of the terms in the Moon's motion due to the actions—direct and indirect—of the planets, and it is of interest to compare the results with those of previous investigators. There are two extensive lists in print: one given by Radau* in 1892, and the other by Newcomb† a few months ago. Both of these give only the terms in longitude, and I shall therefore limit the results set forth here for discussion to these terms, reserving the complete list for a memoir, containing also my methods and the details, which I hope to present to the Society at a later date. I have also omitted here the secular terms and those which depend solely on the lunar arguments, since the results for them are not in doubt, and since they will ultimately be combined with terms arising from the figure of the Earth. Further, I only consider quantities of the first order relative to the planetary masses, while Newcomb has included the mutual perturbations in his values for the direct action. But the latter do not affect any of the terms except one or two of long period, mentioned in No. 10 below.

The great majority (about 400) of the terms are of short period,

* "*Recherches concernant les Inégalités planétaires du mouvement de la Lune*," *Paris Obs. Ann. (Mém.)*, vol. xxi.

† "Investigation of Inequalities in the motion of the Moon produced by the action of the planets," *Carnegie Inst. Publ.*, 72.

and are most conveniently added to the true longitude. The terms of very long period are best added to the mean longitude, and they are so set down here; but the changes in the other elements for these long period terms are not given, since they can be exhibited as simple ratios to the mean longitude portions. Radau adds these long period terms to the true longitude, and I have therefore dropped the corresponding short period terms which he gives.

In his final results, Newcomb drops coefficients less than $0''.003$. I have done the same in transcribing from my manuscripts, except occasionally where a smaller coefficient comes in the middle of a long series, or where Newcomb or Radau give a coefficient greater than $0''.002$. All my coefficients have been computed to $0''.001$ except those of a few long period terms in δw_1 ; these exceptions are noted by a comma placed before the zero, which is set down as the last figure in order not to break the continuity of the representation.

2. The notation for the arguments of the tables is as follows:—

$l, 2D, F$, the mean anomaly, the argument of the
"variation" and the principal argument
of the latitude of the Moon.

V^* , the Moon's true longitude.

w_1 , the Moon's mean longitude.

Q, V, T, M, J, S , the mean longitudes of Mercury, Venus, the
Earth, Mars, Jupiter, and Saturn.

$h = D - F + T$, the mean longitude of the Moon's node.

The results are arranged in six columns. The first pair, headed B, contains the coefficient and angle as computed by myself; the second pair, headed N, those of Newcomb; the third pair, headed R, those of Radau. The coefficients are given in units of $0''.001$. The angle has, except in δw_1 , been divided into four parts for convenience: ϕ contains the lunar arguments l, D, F, h ; α , the constant part of the angle so taken that the coefficients are all positive; j, j' the multiples of $T - P$ and T (or j, j'' the multiples of $P - T$ and P) so that j'' (or j''') represents the order of the term with reference to the eccentricities and inclination of the Earth and planets. The values of j'' (or j''') and ϕ , once given, are understood to be the same for every following term until new values are set down. The differences between the results of Newcomb and myself are marked with the letters E_1, E_2, E_3, U whenever they exceed $0''.020$, and they are considered in No. 8 below.

* An italic capital to distinguish from the mean longitude of Venus.

$$\delta V = +0''.001 \text{ C sin } \{ \phi + j(T - V) + j''T + a \}, \text{ Venus.}$$

$\phi=0$		B		N		R		
j	j'	C	a	C	a	C	a	
1	0	822	0°0	882	0°1	860	0°	E ₁
2		307	179°8	401	179°3	283	180°	E ₁
3		42	359°3	37	1°6	65	0	
4		46	0					
5		33	0					
6		24	0					
7		17	0					
8		12	0					
9		8	0					
10		6	0					
11		4	0					
21		5	0					
-2	1	10	254					
-1		16	84	14	85			
1		42	82	55	81°5			
2		348	272°9	354	276°2	348	275	
3		176	271°7	197	272°2	181	272	E ₁
4		2	82					
5		4	271					
6		6	272					
7		4	272					
-1	2	3	27					
1		5	25					
2		3	33	5	34			
3		92	199°0	82	208°0	80	198	
4		26	204					
5		9	17					
6		4	207					
5	3	26	114	23	112			

${}^2D$		B		N		R		
j	j'	C	α	C	α	C	α	
-11	0	3	0		0		0	
-10		5	0					
-9		6	0					
-8		8	0					
-7		8	0					
-6		11	0					
-5		11	0					
-4		8	0					
-3		34	180.0	56	359	32	180	E_1
-2		36	0.0	24	0.0	48	0	
-1		23	180	8	180	30	180	
1		99	0.0	96	0.0	105	0	
2		136	179.5	143	178.8	148	180	
3		13	178	35	180	23	180	E_1
4		4	180					
18		3	0					
1	I	3	232					
2		40	271.0	102	271.8	46	270	E_2
3		37	271.5	92	272.4	44	270	E_2
4		5	89					
20		10	273					
-11	-I	3	78					
-10		4	78					
-9		4	78					
-8		4	78					
-7		5	78					
-6		5	78					
-5		11	82					
-4		7	78					
-3		2	288	5	268.2	12	270	
-2		19	271	49	267.8	28	270	E_2
1		9	98					
2		13	281					
3		3	281					

² D		C	B	C	N	C	R	α
<i>j</i>	<i>j'</i>		<i>a</i>					
3	2	11	199°	26	199°7			°
4		3	202					
5		4	20					
- 6	- 2	6	162					
- 3		15	342	35	339°8			
- 2		3	7					
15		4	151					
<i>l</i>								
- 7	0	4	180					
- 6		5	180					
- 5		6	180					
- 4		6	180					
- 3		8	180	6	180	10	180	
- 2		61	0°0	71	0°6			
- 1		129	180°0	149	180°0	66	180	E ₁
1		152	0°0	166	0°0	80	0	
2		48	180°0	93	180			E ₁
3		127	180°0	8	0	138	180	E ₁
4		11	180					
21		30	0			56	0	
- 2	I	4	258					
- 1		1	79	4	80			
1		8	75	3	80			
2		46	271	82	273°1			E ₁
3		40	272	41	271°8			
4		5	272					
5		4	92					
23		6	272			12	273	
- 3	- I	32	268	37	267°0			
- 2		46	264	80	266°5			E ₁
- 1		9	104	3	100			
1		3	102	4	280			
2		7	282					
3		7	280					

<i>l</i>		B		N		R	
<i>j</i>	<i>j'</i>	C	<i>a</i>	C	<i>a</i>	C	<i>a</i>
3	2	14	210°	23	201°		°
4		4	205				
5		3	19				
6		16	198				
- 5	- 2	3	161				
- 4		4	336	22	336		
- 3		15	331				
5	3	4	115	5	115		
- 5	- 3	4	65	5	65		
<i>2D-l</i>							
- 10	0	3	180				
- 9		6	180				
- 8		8	180				
- 7		13	180				
- 6		22	180				
- 5		38	180				
- 4		83	180°0				
- 3		658	180°0	646	179°8	681	180
- 2		137	0°0	142	0°0	192	0
- 1		13	180	25	180	42	180
1		133	0°0	146	0°0	143	0
2		157	179°6	174	180°0	172	180
3		14	178	14	180	18	180
4		3	180				
18		11	0			12	0
- 2	1	3	259				
- 1		3	270				
2		65	271°4	71	269°5	70	270
3		49	271°9	53	273°7	50	270
4		5	90				
20		126	273°0			140	272

$2D-l$ $j \quad j'$	C	B a	C	N a	C	R a
- 8 - I	3	269°				
- 7	5	269				
- 6	8	269				
- 5	24	269	25	268		
- 4	30	89				
- 3	0	...			13	270
- 2	38	268	41	264°0	45	270
I	9	101				
2	13	281				
17	3	253				
3 2	19	200	21	202	22	198
4	4	202				
5	4	19				
- 7 - 2	3	340				
- 6	79	162°6	83	161°3	82	163
- 5	4	165				
- 3	24	340	26	337	27	342
- 2	3	6				
15	25	151				
5 3	4	114				
12 - 4	33	303				
$2D+l$						
- 5	3	0				
- 3	4	0				
- 2	4	0				
I	11	0	6	180		
2	15	180	9	0		
3	3	180				
2 I	5	271				
3	4	271				
- 2 - I	3	269				

$2l$		B		N		R	
j	j'	C	α	C	α	C	α
- 2	0	5	0				
- 1		10	180				
1		10	0				
2		6	180				
3		9	180				
2	1	3	273				
- 2	- 1	3	267				
$2l-2D$							
- 2	0	7	0	9	0		
- 1		5	180	7	180		
2		3	180	8	180		
3		73	180.0	72	180.0	75	180
4		3	0				
4	1	4	92				
- 20	- 1	7	267				
- 3		3	268				
- 2		3	268	3	261		
6	2	62	17.4	62	17.5	63	17
$4D-l$							
1	0	6	180				
2		7	0				
$3l-2D$							
3	0	3	180	3	180		
6	2	5	18	3	26		
$2l-4D$							
3	0	8	0				
$-4D$							
3	0	7	0				
$-D$							
3	1	5	273			31	91.4
$l-D$							
3	1	11	273			48	91.4
$2l+2D$							
2	0	0		3	0		

h		B		N		R	
j	j'	C	a	C	a	C	a
- 5	- 2	3	255°		°		°
- 4		9	255				
- 3		16	75			16	256
- 2		5	75				
$h+2F$							
- 2	- 3	3	75				

$$\delta w_1 = +0''.001 \text{ C sin } (\phi + a).$$

ϕ	B		N		R		
	C	a	C	a	C	a	
13T-8V	237	313°9	247	316°1	250	318°7	
l+16T-18V	1449,0	151°0	1477,0	150°1	1442,0	150	E_g
l+3T-10V	35,0	33					
l+29T-26V	108	112°0			110	121	
2F-2D+6T-5V	54	270°0			60	269	
3l-2D+24(T-V)	10	0			12	0	
D+l-F+17T-18V	8	75					
F+24T-23V	3	285			8	0	
D+12T-15V	13	262			16	261	
D+25T-23V	13	190			19	194°5	
3D-2F+18T-18V	2	272			8	0	
2D-2F-l-22T+23V	< 10				4	284	
D-2l-20T+21V	0				3	288	

$$\delta w_1 = +0''.001 \text{ C sin } (\phi + a), \text{ Mercury.}$$

Q-4T	3	239°		°
2D-l-3Q+T	75	105	57	150
2D-l-4Q+5T	3	113	6	105
l-2F+4Q-3T	3	113	7	105

$$\delta V = +0''.001 \text{ C sin } \{\phi + j(M - T) + j''M + a\}, \text{ Mars.}$$

$\phi = 0$		B		N		R		
j	j''	C	a	C	a	C	a	
1	0	11	180°	11	174°	14	180°	
2		195	180°·2	224	180	228	180	E ₁
3		14	357	14	0	16	0	
4		5	349					
- 3	1	6	260					
0		0		30	82			U
1		327	224°·4	372	229°·2	422	229	E ₁
2		38	212°·4	42	213°·6	53	213	
3		48	212°·5	47	212°·9	55	212	
4		10	331					
2	2	93	244°·8	95	249°·8	112	247	
3		20	245	30	120			U
4		14	244					
5		6	62					
3	3	16	277					
4		13	276					
5		6	275					
6		3	94					
2D								
- 2	0	5	0	6	0	16	0	
1		4	180					
2		44	181	50	180°·0	59	180	
3		5	0					
1	1	23	224	52	225°·0	30	224	E ₂
2		6	212					
3		8	214	28	214			E ₂
4		3	37					

^{2D} <i>j j''</i>		B <i>C α</i>		N <i>C α</i>		R <i>C α</i>	
- 5 - 1		3	149°				•
- 3		1	327	5	333		
- 2		3	328				
- 1		23	317	58	316·8	34	315 E ₂
3		3	280				
2 2		5	244				
3		4	244				
4		4	246				
- 2 - 2		8	297				
<i>l</i>							
- 3 0		3	180	3	180		
- 2		38	0	43	0·0		
- 1		4	0	4	0		
1		5	180	4	180		
2		43	180	48	180		
3		3	0	3	0		
4		3	180				
0 1		0		3	80		
1		73	223·3	55	219·9	35	230
2		10	212	9	215		
3		13	213	10	215		
5		8	198				
- 3 - 1		9	330	10	324		
- 2		8	327	9	324		
- 1		74	306·3	55	318·9	36	310
0		0		3	99		
2 2		17	245	17	244		
3		5	245				
4		3	244				
6		6	63				
- 4 - 2		3	296				
- 3		5	295				
- 2		18	295	17	296		
26		0				12	306

$2D-l$		B		N		R	
j	j''	C	α	C	α	C	α
- 5	0	3	180°		°		°
- 4		20	182			19	180
- 3		5	0	6	0		
- 2		13	0	17	0	34	0
- 1		3	0	4	0	7	0
1		8	180	5	180	9	180
2		61	181	66	180'0	73	180
3		5	353	3	0		
1	1	31	220	41	226'4	47	223
2		11	212	9	215		
3		14	214	13	215		
4		3	27				
- 6	- 1	3	149				
- 5		43	162			42	148
- 4		3	329				
- 3		3	327	5	333		
- 2		6	328	6	324		
- 1		35	320	43	316'9	51	317
3		4	280				
2	2	11	244	9	244		
3		6	244				
4		5	245				
- 6	- 2	33	298				
- 3		3	296				
- 2		14	297	11	294		
$2D+l$							
2	0	6	180	4	0		
1	1	3	82				
- 1	- 1	3	93				
$2l$							
- 2	0	3	0				
2		3	180				
1	1	3	232				
- 1	- 1	3	308				

$2l-2D$		B		N		R	
j	j''	C	α	C	α	C	α
-2	0	4	0	4	0		0
5	1	17	209			17	212
6	2	18	244				

$$\delta w_1 = +0''.001 \text{ C sin } (\phi + \alpha), \text{ Mars.}$$

ϕ	B		N		R	
	C	α	C	α	C	α
8M-4T	.3	310				0
9M-5T	8	305				
11M-6T	6	335				
13M-7T	6	19				
15M-8T	26	43				
17M-9T	4	63				
4D-3l+25M-23T	4.0	67				
h+2M-T	17	165			19	164
D-l-4M+3T	<10				3	153

$$\delta V = +0''.001 \text{ C sin } \{ \phi + j(J-T) + j''J + \alpha \}, \text{ Jupiter.}$$

$\phi=0$		B		N		R		
j	j''	C	α	C	α	C	α	
1	0	643	178.8	741	178.8	646	180	E ₁
2		187	359.6	242	359.8	196	0	E ₁
3		10	7	9	6	12	0	
-3	1	6	257	6	80			
-2		18	274	16	275			
0		87	289.9	183	283.0	139	279	U
1		165	241.5	193	242.7	172	242	E ₁
2		52	352.0	42	77.7	49	352	U
3		4	355					
-1	2	10	250					
0		5	324					
1		25	238	24	83			U
2		6	344					
1	3	3	230					

² D		B		N		R'		
<i>j</i>	<i>j''</i>	C	<i>a</i>	C	<i>a</i>	C	<i>a</i>	
-3	0	4	0	18	180		0	E ₂
-2		70	180°0	18	0	54	180	E ₁
-1		33	1°1	16	0	45	0	
1		167	178°5	168	177°9	175	180	
2		85	359°2	92	359°3	88	0	
3		7	13	74	2°3			U
0	1	27	349	36	350°5	32	350	
1		35	236°5	89	237°4	39	238	E ₂
2		15	352	42	354°5			E ₂
-2	-1	30	8	7	9			E ₁
-1		6	313	15	311			
0		33	184	40	181°0	38	189	
2		9	273	38	272°0			E ₂
3		6	102	19	105			
1	2	5	236	10	240			
2		3	345					
0	-2	3	200					
1		6	110					
<i>l</i>								
-3	0	1	170	4	180			
-2		36	180	45	180	12	180	
-1		144	1°0	140	1°6	36	0	
1		158	179°0	163	178°6	48	180	
2		190	180°0	64	0°0	165	180	E ₁
3		5	21	3	0°0			
-2	1	6	274	6	270			
0		62	282°3	38	7°5			E ₁
1		39	242	36	268°4			
2		96	352°5	10	351	65	353	E ₁

$\begin{matrix} l \\ j \quad j'' \end{matrix}$		$\begin{matrix} B \\ C \quad a \end{matrix}$		$\begin{matrix} N \\ C \quad a \end{matrix}$		$\begin{matrix} R \\ C \quad a \end{matrix}$	
-2	-1	7	188°	10	189°		
-1		35	298	32	278°6		
0		63	257°2	38	172°5		U
2		6	273	6	270		
3		8	286				
0	2	7	326				
1		5	238	3	206		
2		4	343				
-1	-2	5	302	3	334		
0		7	214				
$2D-l$							
-4	0	4	180				
-3		22	182	18	0		U
-2		1137	180°3	1140	180°2	881	180
-1		51	0°9	64	0°0	101	0
1		211	178°4	230	179°0	220	180
2		89	359°2	98	0°7	89	0
3		6	14	30	1°9		U
-3	1	5	261				
-2		13	310	18	294		
0		56	5°5	60	353°4	60	351
1		46	237°0	48	238°7	49	236
2		20	352	21	353		
-3	-1	6	187				
-2		436	7°5	445	7°5	316	7
-1		18	296	19	311	25	304
0		60	174°2	62	182°2	63	184
2		16	273	18	276		
3		7	102				
1	2	6	237	4	237		
2		3	344				
-2	-2	5	19			9	16
1		3	291				

$2D+l$		B		N		R		U
j	j''	C	a	C	a	C	a	
-2	0	3	180°		°			
-1		5	1					
1		21	178	11	0			U
2		7	359	6	180			
0	1	4	353					
1		5	237					
2		3	352					
0	-1	4	182					
2		3	273					
$2l$								
-2	0	3	180					
-1		11	2					
1		12	178					
2		10	180					
0	1	5	293					
1		3	239					
-1	-1	3	301					
0		5	247					
$2l-2D$								
-2	0	5	180	4	180			
-1		11	2	11	0			
1		3	0					
2		240	179.9	256	180.0	206	180	
0	1	0		4	350			
2	1	284	172.5	258	172.3	194	173	U
-1	-1	2	302	3	298			
0	-1	2	186	4	189			
2	2	3	163			2	164	
$4D-l$								
-2	0	7	180					
1		9	358					
2		5	179					
-2	-1	3	7					
$3l-2D$								
2	0	7	180	10	180			
2	1	5	172	15	257			

$2l-4D$		B		N		R	
j	j''	C	a	C	a	C	a
2	0	9	0°		0°		0°
2	1	7	173				
h							
0	1	4	81			4	75
0	-2	0				11	277

$$\delta V = +0''\cdot001 \text{ C sin } \{\phi + j(S - T) + j''S + a\}, \text{ Saturn.}$$

$\phi=0$		B		N		R	
j	j''	C	a	C	a	C	a
1	0	42	179°6	40	180°0	38	180°
2		8	0	8	0		
0	1	21	273	51	278°9		
1		13	257	11	255		
0	2	3	297				
U							
$2D$							
1	0	10	180				
2		5	0				
0	1	4	270				
1		3	257				
0	-1	4	255				
l							
-1	0	6	0	8	0		
1		10	180	8	180		
2		3	180				
0	1	12	263	4	275		
1		3	257				
-1	-1	3	283				
0		12	277	4	265		
$2D'-l$							
-2	0	19	180				
1		14	180				
2		4	0				
0	1	6	271				
1		3	257				
-2	-1	5	271				
0		6	267				

3. In order that the discussion of the various differences may be made clear, it is necessary to state certain facts in the theory of these inequalities which have considerable effect on the final results, and on the methods of obtaining them.

The disturbing function R is of the form—

$$R = \Sigma \{ A_0 \cos Q + A_1 \cos (l + Q) + A_2 \cos (2D - l + Q) \\ + A_3 \cos (2D + l + Q) + A_4 \cos (2D + Q) \\ + A_5 \cos (2D - 2l + Q) + \dots \},$$

where Q here and throughout this discussion is an angle depending only on the solar and planetary arguments, and where the summation sign refers to the different arguments Q .

Let w_1 be the Moon's mean longitude, and c any one of the other five lunar elements. When the value of R has been substituted in the equations of variations and the latter solved we obtain—

$$\delta w_1 = \Sigma \{ B_0 \sin Q + B_1 \sin (l + Q) + \dots \},$$

$$\delta c = \Sigma \left\{ C_0 \frac{\sin}{\cos} Q + C_1 \frac{\sin}{\cos} (l + Q) + \dots \right\},$$

the term in δc being a sine or cosine according as c is or is not an angular element.

The values of the variations of the elements have to be substituted in—

$$\delta V = \delta w_1 + \left(\frac{dV}{dw_1} - 1 \right) \delta w_1 + \frac{dV}{dc} \delta c.$$

Now V consists of a non-periodic term w_1 and periodic terms, say,

$$V = w_1 + v_1 \sin l + v_2 \sin (2D - l) + v_3 \sin (2D + l) + \dots,$$

and from this we obtain easily the values of the derivatives of V . The terms which arise from the substitution of the variations of the elements in δV may therefore be divided into two classes. The first of these, which I have called elsewhere* the *primary* terms, consists of those terms in δV which arise from the variation of the non-periodic term; the *secondary* terms are those arising from the substitution in the remaining, that is, the *periodic* terms of V . Thus the values of δw_1 constitute the *primary* terms, and all the other portions of δV the *secondary* terms.

Now the primary terms have the same arguments as the terms in R , that is, the arguments

$$Q, l + Q, 2D - l + Q, \dots,$$

while the secondary terms have the arguments

$$Q \pm l, Q \pm (2D - l), Q \pm (2D + l), \dots \\ l + Q \pm l, l + Q \pm (2D - l), l + Q \pm (2D + l), \dots \\ \dots$$

* *The Inequalities in the motion of the moon due to the direct action of the planets*, Pitt Press, Cambridge, 1907.

the arguments arising from the derivatives of V being enclosed in brackets. It is therefore obvious that the arguments $Q, l+Q, \dots$ which are present in the primary terms will also appear amongst the secondary terms; they are due to the separation of a product of a sine and a cosine into the sum or difference of two sines.

4. The first result to be noted is the fact that *the sum of the secondaries with arguments $l+Q-l, 2D-l+Q-(2D-l), \dots$ that is, those having the argument Q , is very small compared with the coefficient of the primary with argument Q .** In many cases these secondaries are of the same order of magnitude as the primary of the same argument, the chief of them in general being those arising from the two arguments just written down. This theorem is a consequence of the method of the variation of arbitrary constants. If we had adopted the straightforward method of integrating the original equations for the Moon's motion with the additional value of R due to planetary action, the terms independent of l in the coordinates arising from the terms in R which contain l , and which therefore have the factor e (the lunar eccentricity), would have possessed the factor e^2 , while the principal terms due to $R=A_0 \cos Q$ would not have this factor; the former terms must therefore be quite small.

This fact constitutes a useful test of the general accuracy of a large portion of the work, and it was satisfied in all cases as closely as could be expected. It is illustrated in the following table for a few terms of the indirect action of Venus in longitude. The first column gives the arguments of the primaries (that is, of the terms in R), and in line with them are the resulting secondaries for the values of ϕ which stand at the head of each column. The last line but one gives the sums of these secondaries, and the last line the coefficients of the primaries. The sums of these two lines constitute the complete coefficients for the indirect action. The constant angle to be added to each argument is omitted, as it is (in these and in most cases) very nearly the same for each of the numbers in a given column.

$\delta V = 0'' \cdot 001 C \sin(\phi + \alpha), \phi = i'T - i''V$, Venus. Values of C .

Arg. of primary.	T-V	T-2V	2T-2V	3T-2V	3T-3V	4T-3V	5T-3V
$\phi + l$	- 47	- 7	- 92	+ 18	- 13	+ 21	+ 5
$\phi - l$	- 8	...	+ 21	+ 7	+ 4	- 3	+ 7
$\phi + 2D - l$	+ 42	+ 6	+ 70	- 20	+ 9	- 18	- 5
$\phi - 2D + l$	+ 18	...	+ 3	- 8	...	- 3	- 7
$\phi + 2l - 2D$	+ 1	...	...	...	+ 1	...	...
$\phi - 2l + 2D$	+ 2	...	+ 2	...	...	...	...
Sums	+ 8	- 1	+ 4	- 3	+ 1	- 3	0
ϕ	- 352	- 14	- 507	+ 275	- 51	+ 155	+ 129

* This result was not noticed until the computations had been practically completed, when the approximate vanishing of the sums of the secondaries in all cases pointed to a general theorem.

5. At first sight it would appear that the arguments of the two terms formed, for example, from sums of products like $\sin(l+Q)\cos(2D-l)$, that is, $l+Q \pm (2D-l)$, would have coefficients in δV of the same order of magnitude. It is not so whenever the arguments in R and V contain l , unless the argument in R is of very long period (greater than a score or so of years). This fact might have been predicted from considerations of the same nature as those in the previous paragraph. For example, an argument $2l+Q+(l) = Q+3l$ would not be expected from a direct method of treatment to have a coefficient nearly so large as that of $2l+Q-(l) = Q+l$. The latter is frequently sensible, while there are no terms with the former argument.

This arises, again, from the peculiarities of the method of the variation of arbitrary constants. In the volume referred to above I have shown directly from the method that *if a term of R contains il in its argument ($i=1, 2, 3, \dots$) and V a term $i'l$ ($i'=1, 2, 3, \dots$), then the secondary arising from the sum of these arguments is always very small compared with that arising from the difference, unless the period of the primary is very long.* This result was proved by inserting the terms due to δl , δe , as given by the equations of variations, in δV . It enabled me to abbreviate the computations by some twenty or more per cent., since it rendered the separate computation of δe unnecessary except in a few easily recognised cases.

6. One other fact should be noted. Although the coefficient of the evection (arg. $2D-l$) is of order $m(1/13)$ compared with that of the principal elliptic term (arg. l), the numerical factors which it contains are so large that in certain functions of the lunar co-ordinates it becomes equal to or of more importance than the principal elliptic term. It is therefore never safe to neglect the former terms unless we know that the latter are quite insensible. This shows most strongly in the arguments $l+\phi$ in R for the indirect action. The terms which multiply $\delta V'$ (the portion of the Earth's longitude due to the action of the planets) are chiefly those due to the evection, while those which multiply $\delta r'$ (the planetary portion of the Earth's radius vector) are due to the evection and the principal elliptic term. Yet the former portions in several cases are larger than the latter.

7. *The differences $B-R$.* Each of Radau's coefficients depending on the action of Venus is to be diminished by $1/51$ of its amount on account of the difference between the value for the mass of Venus adopted by him and that adopted by Newcomb and myself. Each coefficient due to Mercury is to be diminished by $7/60$ of its amount for my adopted value of the mass of that planet.

The equations of variations used by Radau appear to require a correction which would increase each coefficient in the ratio $96:95$, as I have shown in the volume previously referred to. This is confirmed by Newcomb's results, for he obtains values for the principal terms the same as those which I found in the volume quoted.

In the primary terms with arguments independent of the lunar angles, Radau omits the portions due to the term of R which depends on $r^2 \cos 2(V - T)$. These portions constitute about one-eighth of the principal portions for the indirect action, and his coefficients for this portion should therefore be diminished in the ratio 7:8. The corresponding correction for the direct action is smaller, and its amount depends on the particular term under consideration.

In the short period terms of the disturbing function for the indirect action of the form $l + \phi$ he omits the portions multiplied by $\delta V'$. These, as stated in No. 6 above, constitute a considerable fraction of the whole. The terms *proportionately* most affected are those with this argument, but the *differences* for the terms of argument $l + \phi - (l)$ are much larger, for the latter (secondary) terms are about three times the former (primary) terms.

The discovery of the causes of divergence between the results of Radau and myself was rendered difficult owing to the fact that, although his secondary inequalities for the Venus terms of argument ϕ are given separately, and are in several cases quite large when the theorem of No. 4 above shows that they ought to be quite small, yet his final coefficients very nearly agreed with mine. As a matter of fact, I have found, from an examination of the separate portions of my results, that these corrections very nearly balance one another in the case of Venus, though they do not do so with the other planets, nor do they in the case of Venus with arguments other than ϕ .

I have compared most of Radau's individual results with mine, and find a close agreement for nearly all those portions which he has computed. For the direct action this agreement was particularly useful, since my coefficients are obtained from formulæ and methods radically different from his, though reducible to them by algebraical processes.

Two other differences should be noticed, namely, those in the terms with arguments $-D + 4T - 3V$ and $l - D + 4T - 3V$. These arise from the fact that the indirect action is a little greater and of the opposite sign to the direct action; Radau only takes the latter into account.

8. *The differences B - N.* These have been marked with the letters E_1 , E_2 , E_3 (explained), and U (unexplained), whenever they exceed $0''.020$.

Those marked E_1 are due to the fact that Newcomb substitutes the variations of the elements in the value of V limited to the three terms

$$V = w_1 + 2e \sin l + \frac{5}{4}e^2 \sin 2l,$$

instead of taking the complete value; he adds that "in nearly or quite all cases we may drop terms of the second order in e ."

This third term of V in reality only produces a few secondaries with coefficients greater than $0''.020$, and these arise in the form $2D - 2l + \phi + (2l) = 2D + \phi$; but there are many greater than

0".003, the adopted limit below which coefficients were dropped. The latter arise also in the forms $l + \phi - (2l)$ and $2D - l + \phi + (2l)$.

A much more extended and larger set of differences arises, however, from the neglect of the evection in the expression for V . The majority of the differences E_1 due to this cause will be found in the combinations—

$$\begin{aligned}\phi &\pm (2D - l), \\ l + \phi + (2D - l) &= 2D + \phi, \\ 2D - l + \phi - (2D - l) &= \phi, \\ 2l - 2D + \phi + (2D - l) &= l + \phi,\end{aligned}$$

the last being of importance only when $2l - 2D + \phi$ is of long period of the order of ten years, and the first quite rarely. Hence the final coefficients of terms with arguments ϕ , $2D + \phi$ will be chiefly affected, and a very few in those with arguments $l + \phi$, from this cause. All those differences which may be nearly or altogether explained by the neglect of this and other terms in V are marked E_1 . I assume that the terms not present in Newcomb's list have not been computed.

In order to examine the differences E_2 a closer inspection is necessary. On referring to Newcomb's tables for the variations of the elements (p. 154 of his work), I find that under the argument $2D$ he has only the terms containing $(V - T)$ and $2(V - T)$ for Venus. There are, however, several other arguments with coefficients of the same order of magnitude as these, and the same fact appears to a smaller extent with the other planets. All differences explained by this cause have been marked E_2 .

The small difference E_3 in the great long period term due to Venus scarcely needs explanation. Newcomb has included the portion (less than 0".10) due to the mutual perturbations of Venus and the Earth, and he states that his method indicates a possible error of the order 0".10. I exclude the former part, but the maximum error by my method should be less than 0".05.

Of the differences U there are two in Mars, eight in Jupiter, and one in Saturn. The most important is that in Jupiter with argument J ; I have compared the several portions of any value for the primary with that of Radau, and the results (after the corrections noted in No. 7 above have been made) agree for the indirect action, but I have been unable to obtain his result for the direct action from the algebraical formulæ which he gives. The causes for the remaining differences I have not been able to trace, and must leave the values of those coefficients an open question.

9. In working out the terms produced by planetary action, the valuable memoir of M. Radau has been available for comparison of results at almost every stage, and has materially assisted in the prevention of errors made during the course of a piece of work of great complexity, though not of great difficulty when once the theory had been put into final shape. The work of Professor Newcomb only appeared when I had finished the greater part of

the computations, and would in any case not have been available for detailed comparison without much labour, owing to the complete difference between his method and mine. He combines the direct and indirect actions at the earliest opportunity. I have kept them separate until each was fully completed. The comparison between his results and mine revealed one error in my work which affected the primaries due to the terms with argument ϕ in R by about 5 per cent., and a few of the small secondaries from these terms about twenty per cent.; an error in the equations of variations had almost no effect. These errors have, of course, been corrected in the results given above.

10. Like Professor Newcomb, I have also made an examination of the inequalities of the second order relative to the planetary masses, and have so far found nothing that could sensibly affect the motion of the Moon. I have found an additional portion to the term with argument $l + 3T - 10V$ of the order of $0''.2$, but the period of this term is so long that it would scarcely affect the observations within the degree of accuracy at present obtainable. The motion of the node of Venus also affects the term with argument $l + 16T - 18V$ by a quantity of the same order of magnitude. The terms of the second order in the Sun's motion, as given in Newcomb's Tables of the Sun,

$$\delta V' = -0''.265 \cos(4M_1 - 7T_1 + 3V_1) - 0''.021 \sin(4M_1 - 7T_1 + 3V_1) \\ + 3''.76 \cos(3J_1 - 8M_1 + 4T_1) + 5''.18 \sin(3J_1 - 8M_1 + 4T_1)$$

(where the suffix denotes the mean anomalies of the planets instead of the mean motions), or

$$+ 0''.266 \sin(1^\circ.190t + 31^\circ.8) \quad (\text{period 300 years}) \\ + 6''.40 \sin(0^\circ.202t + 231^\circ.2) \quad (\text{period 1780 years}),$$

will produce inequalities with the same arguments due to indirect action having the approximate coefficients

$$-0''.04 \quad \text{and} \quad -0''.9$$

respectively. These results are sufficiently accurate for tabular purposes, but I shall give a more complete computation, with an examination of the whole effect of the terms of the second order.

New Haven, Conn. U.S.A.:
1907 November 26.

Postscript.—Since this paper was sent in, I have computed the terms due to the motion of the ecliptic and have discovered a few new inequalities containing the arguments of the planets. The most important is one with a coefficient $0''.21$ due to Jupiter, and having a period of 280 years.

1908 January 10.

Note on the Single Equation which comprises the Theory of the Fundamental Instruments of the Observatory. By Sir Robert Ball, LL.D., F.R.S.

We may conceive a generalised astronomical instrument of which the essential parts are as follows:—

There is a fundamental axis, which we shall distinguish as axis I. It is capable of rotation in fixed bearings, and to it is attached an index which points to a reading R on a fixed graduated circle A . Axis I passes through the centre of A , and is normal to the plane of A .

Axis II is capable of rotation in bearings fixed on axis I. A second graduated circle B is attached to axis II which passes through the centre of B , and is normal to its plane. The reading of B is R' , as shown by an index rigidly attached to I. It may be observed that an index parallel to the intersections of the planes of A and B will serve for reading both circles, and the geometry of the question is simplified by employing this index.

It is necessary to distinguish between the two poles on the celestial sphere which are defined by the plane of a graduated circle. From one of these poles the graduation would appear to increase clockwise. From the other pole the graduation would appear to increase anti-clockwise. It is the latter pole which we shall here employ. The angle between I and II is the angle ($\nless 180^\circ$) between the poles of A and B . We shall express it by $90^\circ - q$.

The telescope is rigidly attached to axis II, and when the optical axis of the telescope is directed to a star, the arc ($\nless 180^\circ$) from that star to the pole of B is also a constant of the instrument. We shall denote it by $90^\circ + r$.

The semiplane through axis II and that half of the telescope which contains the objective, cuts B at the graduation we shall term Δ .

Let R_1 , R_1' and R_2 , R_2' be the readings of the instrument when directed successively to stars S_1 and S_2 , with celestial co-ordinates α_1 , δ_1 and α_2 , δ_2 . These co-ordinates may be altitude and azimuth, or right ascension and declination, or latitude and longitude, or any system in which the fundamental circles are rectangular. Then the equation we desire is obtained by equating two different

expressions for the cosine of the arc between S_1 and S_2 , and it is as follows:—

$$\begin{aligned}
 & \sin \delta_1 \sin \delta_2 + \cos \delta_1 \cos \delta_2 \cos (a_1 - a_2) \\
 &= + \sin^2 q \sin^2 r \\
 & \quad + \cos^2 q \sin^2 r \cos (R_1 - R_2) \\
 & \quad + \cos^2 q \cos^2 r \sin (R_1' - \Delta) \sin (R_2' - \Delta) \\
 & \quad + \cos^2 r \cos (R_1 - R_2) \cos (R_1' - \Delta) \cos (R_2' - \Delta) \quad (i) \\
 & \quad + \sin^2 q \cos^2 r \cos (R_1 - R_2) \sin (R_1' - \Delta) \sin (R_2' - \Delta) \\
 & \quad + \cos^2 r \sin q \sin (R_1 - R_2) \sin (R_1' - R_2') \\
 & \quad + \cos q \sin r \cos r \sin (R_1 - R_2) \{ \cos (R_2' - \Delta) - \cos (R_1' - \Delta) \} \\
 & \quad + \sin q \cos q \sin r \cos r \{ \cos (R_1 - R_2) - 1 \} \{ \sin (R_1' - \Delta) \\
 & \quad \quad \quad + \sin (R_2' - \Delta) \}
 \end{aligned}$$

By assigning suitable values to q and r , this formula can be made to apply to the following astronomical instruments:—the altazimuth, the meridian circle, the prime vertical instrument, the equatorial, and the almucantar. For the meridian circle q and r should be each as near zero as possible, and for the almucantar q is the latitude and r quite arbitrary. The following general proof will show that the complete theory of each of the instruments named must be included in this one formula.

From any such instrument we demand no more than that the two readings R and R' obtained by directing the instrument to any particular star shall enable us to calculate the co-ordinates a , δ of that star free from all instrumental errors.

Let S_1 , S_2 , S_3 be three standard stars of which the co-ordinates are known, and let each of these stars be observed with the generalised instrument with results R_1 , R_1' ; R_2 , R_2' ; R_3 , R_3' respectively. Substituting for each of the three pairs (S_1 S_2), (S_2 S_3), (S_3 S_1) in the typical formula (i), we obtain three independent equations. From these equations, q , r , and Δ can be found. Nor will there be any indefiniteness in the solution, for in each case we may regard these quantities as approximately known, so that to obtain the accurate values of q , r , and Δ we shall have to solve only linear equations. We may thus regard (i) as an equation connecting a_1 δ_1 , a_2 δ_2 , R_1 , R_1' , R_2 , R_2' , and known quantities.

Let S be the star whose co-ordinates a , δ are sought. We write the equation (i) for the pair (S S_1), and substitute their numerical values for a_1 , δ_1 , R_1 , R_1' . We thus have an equation connecting the co-ordinates a , δ of *any* star with its corresponding R , R' and known numerical quantities. When we substitute for R and R' the values observed for S , the formula reduces to a numerical relation between the a and δ of the particular star S . From the pair (S S_2) we find in like manner another quite independent numerical equation involving a , δ . As, however, the equations are not generally sufficient to determine a , δ without indefiniteness, we obtain a third equation from (S S_3). This

equation is not independent of the others, but if we make $x = \sin \delta$, $y = \cos \delta \cos \alpha$, $z = \cos \delta \sin \alpha$, we shall obtain three linear equations in x, y, z which can be solved, and thus α and δ are found without any ambiguity whatever.

All the ordinary formulæ used in connection with the different instruments named can be deduced as particular cases of the general equation (i).

In general, there are no real values of R and R' when the instrument is directed to the pole of circle A . In such a case R would have to be set on one of the imaginary circular points at infinity.

2nd January 1908.

The Perturbations of Halley's Comet in the Past. Second Paper.
The Apparition of 1222. By P. H. Cowell, M.A., F.R.S.,
 and A. C. D. Crommelin, B.A.

In the first paper of this series we identified the comet of October 1301 with Halley's, and found the value $44''.858$ for the mean daily motion at that epoch. We have now completed (with the aid of Mr. F. R. Cripps) the calculation of the perturbations by Jupiter and Saturn for the preceding revolution. As a first approximation, Hind's date (mid-July 1223) was assumed for the preceding perihelion passage, and on this assumption the results were as follows:—

Planet.	Limit of u .	dn .	$d\pi$.	$d\zeta$.
Jupiter	$0^\circ - 90^\circ$	$-''2261$	$-357''$	$-6550''$
„	$90-270$	-0823	-35	$+6001$
„	$270-360$	$+5624$	-183	-353
Saturn	$0-90$	-0619	$+12$	-1784
„	$90-270$	$+0636$	-39	$+1602$
„	$270-360$	-2411	$+15$	$+41$
Sum	...	$+0146$	-587	-1043

Hence mean motion in 1223 = $44''.858 - 0''.015 = 44''.843$

and calculated period = $\frac{1296000'' + 1043''}{44''.843} = 28924$ days.

This indicated 1222 August 15 as the date of the preceding perihelion passage, or 11 months earlier than Hind's date. This is too large a discordance to be possible, so Hind's identification of

the comet of July 1223 with Halley's is erroneous. There was, however, a much more remarkable comet which appeared at the exact epoch indicated by the calculation; and examination shows that the greater part of the statements made concerning it by contemporary writers are quite consistent with its being Halley's, so that the identity is placed beyond reasonable doubt.

The error of the first assumption is so great that it is necessary to recompute the perturbations; this has as yet only been done approximately, the resulting date being 1 day earlier, or August 14; the small difference between this and the preceding result is an illustration of the general proposition that the date of the more remote perihelion passage need only be very roughly known in order to obtain the periodic time correctly.

Pingré's description of this comet is as follows; "En Automne, c'est à-dire aux mois d'Août et de Septembre, on vit une étoile de première grandeur, fort rouge, et accompagnée d'une grande queue qu'elle étendait vers le haut du ciel, en forme d'un cône extrêmement aigu: elle paraissait fort près de la Terre: on l'observa (d'abord) vers le lieu où le soleil se couche au mois de Décembre. Le 15 Août, jour de la première apparition de cette comète (peut-être à Milan) la Lune fut comme morte; elle n'avait plus d'éclat, et elle joignit la comète. On vit ensuite cette comète à l'occident, et même vers le nord, avant la fin du mois d'Août. En Chine on l'observa le 10 Septembre, entre la constellation Kang (les pieds de la Vierge, = ι , κ , λ , θ Virginis), Arcturus et la chevelure de Bérénice: elle disparut le 8 Octobre. Le Père de Mailla dit que les Chinois virent une comète à l'ouest en 1222, à la première lune: c'est sans doute une erreur du copiste, il faut lire à la huitième lune."

The words in parentheses are not part of the original documents, and Pingré's interpolation "d'abord" appears to be erroneous; it is the place where the comet was last seen, not first seen. When the comet extended its tail towards the zenith it must have been nearly vertically above the Sun.

The following is the description in Williams' *Chinese Observations of Comets*:—"In the reign of Ning Tsung, the 15th year of the epoch Kea Ting, the 8th moon, day Kea Woo (= 1222 September 15), a comet appeared in Yew She Te (η , τ , ν Boötis). Its luminous envelope was 30 cubits long. Its body was small, like the planet Jupiter. It was seen for two months. It passed through Te (α , β , γ , ι Libræ), Fang (β , δ , π Scorpii), and Sing (Antares, etc.), and then disappeared." (Williams, in this and other places, erroneously gives ν Boötis instead of ν as one of the components of Yew She Te.)

Most of the above statements are satisfied by the following orbit, in which the longitude of perihelion is that actually derived from the perturbations, and the date of perihelion passage is 8 days later than that indicated above; a perihelion distance somewhat greater than the present value has been used. Hind found that this was also indicated in the return of 1066.

Elements in old System of Elements.

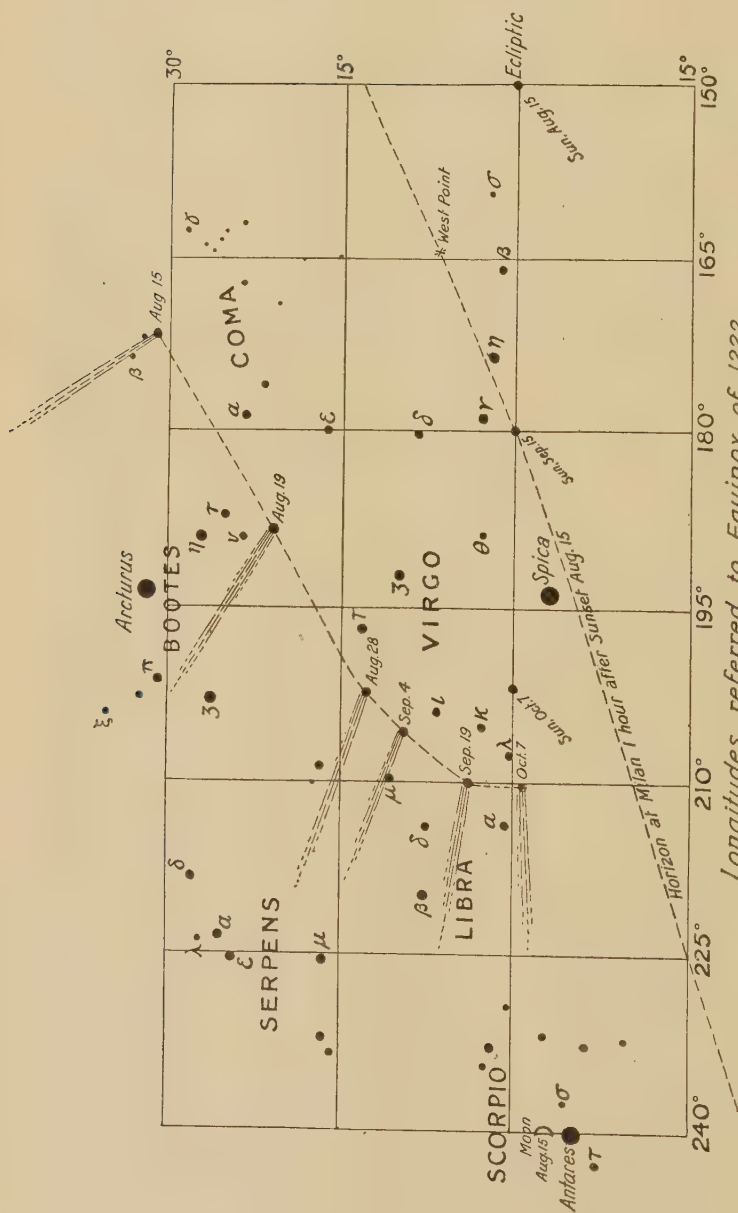
$$\begin{array}{rcl}
 T & = & 1222. \text{ August } 22. \\
 \varpi & = & 296^{\circ}.5 \\
 \Omega & = & 42 \\
 i & = & 16.5 \\
 q & = & 0.67
 \end{array}
 \left. \vphantom{\begin{array}{rcl} T \\ \varpi \\ \Omega \\ i \\ q \end{array}} \right\} \text{Equinox of } 1222.$$

Motion retrograde.

We have had to assume that the date when the comet was first seen in China (in the region η, τ, ν Boötis) should be one lunar month earlier than that given, or August 17 instead of September 15. Our reasons are: (1) it is distinctly stated that the comet was seen for two months in China, the final date being October 8, the position then being in Scorpio; (2) we cannot make this a month later, for in November Scorpio was invisible; (3) further, we are distinctly told that the comet was a splendid object in Europe in mid-August, so that it is most improbable that the Chinese should have missed it till a month later; (4) further, its geocentric motion was at that time so rapid that had it been in Boötes in mid-September it would have been a morning-star on August 15, which it apparently was not.

Another difficulty that we have to surmount is the phrase "*La Lune joignit la comète.*" It was doubtless this phrase that led Pingré to the conclusion that the motion was from south to north, which is in opposition to the Chinese account. Considering the vagueness of all European cometary observations at that epoch, we may look on the phrase as sufficiently satisfied by the fact that the Moon (at her first quarter) and comet were symmetrically placed above the western horizon an hour after sunset, though separated by some 70° . There is one more point to notice. The comet is said to have been near Antares when last seen (October 8). Our chart shows that it was then in Libra, but the head was too near the Sun to be seen; the tail would point to Antares, and, considering its length, it may well have reached that star. It will be seen that we have given the European observations most weight for dates, the Chinese most weight for the track among the stars. This conclusion is in agreement with that reached in the case of other comets, including that of 1301. We have thus shown that the observations are satisfied with no further alteration than they would in any case require to render them self-consistent, and the exact coincidence of date leaves scarcely any room for doubt as to the identity. Pingré was prevented from making this identification by not knowing the wide range through which the period of Halley can alter, and doubtless his inference that the comet moved at first from south to north put Hind off the track.

It is of interest to point out that the revolution 1222-1301 is the longest on record, being 79 years 2 months. It just exceeds that from 1066 to 1145, which was hitherto looked on as the longest. The round now being accomplished is the shortest on



Longitudes referred to Equinox of 1222.

CHART SHOWING THE PATH OF HALLEY'S COMET, 1222.

record, lasting only 74 years $5\frac{1}{2}$ months, a difference of nearly 5 years.

Though the computations for the revolutions 1066-1145, 1145-1222, are still incomplete, enough has been done to make it extremely probable that Hind's identification is correct in each case, though his elements of the apparition of 1066 clearly need modification. They differ more widely from the present elements than perturbations will account for. However, the positions are so vaguely recorded that large alterations in them are possible.

[*Note added January 28.*—Since writing this paper we have done further work on the revolution 1145-1222, the result of which tends to the conclusion that the date of perihelion passage in 1222 was in September rather than August. We are therefore somewhat doubtful whether the date September 15 for the comet being in Yew She Te may not be right after all; if so, the European dates require alteration. We shall return to this point in a subsequent paper. It does not weaken the conclusion as to the identity of the comet of 1222 with Halley's.]

Tables have now been constructed giving the values of the definite integrals for Jupiter and Saturn for the outer half of the orbit. To use them, we have first to find the number of days from perihelion to the extremities of the minor axis, which is done by multiplying the period in days by the number whose log is 8.9825. Hence we find Julian days of passing the points $u = 90^\circ$, $u = 270^\circ$, and then find g' for Jupiter and Saturn by the formulæ—

$$g'_{\text{J}} = \frac{(\text{Julian Day} - 2390645) 360^\circ}{4332.85},$$

$$g'_{\text{S}} = \frac{(\text{Julian Day} - 2388249) 360^\circ}{10760.7}.$$

Then the tables give the values of $\int dn$, $\int d\varpi$, $\int d\zeta$, corresponding to the values of g' at first and third quadrant. The algebraical sum of the quantities has to be taken and added to that deduced from the mechanical quadratures.

The numbers from the tables should be modified as follows:—

Multiply $\int dn$ by the number whose log is $\frac{5}{2}$ [$1.2585 - \log a$], the other columns by the number whose log is [$1.2585 - \log a$], where a is given its value for the revolution under consideration. This does not apply to the constant part of $\int d\varpi$, $\int d\zeta$, which is independent of a .

Table of the definite integrals $\int dn$, $\int d\pi$, $\int d\zeta$, for Jupiter and Saturn between the values 90° and 270° of u , the comet's eccentric anomaly.

g'	Jupiter, $u = 90^\circ$.			Jupiter, $u = 270^\circ$.			Saturn, $u = 90^\circ$.			Saturn, $u = 270^\circ$.		
	$\int dn$	$\int d\pi$	$\int d\zeta$	$\int dn$	$\int d\pi$	$\int d\zeta$	$\int dn$	$\int d\pi$	$\int d\zeta$	$\int dn$	$\int d\pi$	$\int d\zeta$
0	+ '3320	- 306"	+ 5434"	+ '0932	+ 23"	- 125"	+ '0808	+ 38"	+ 1143"	- '0284	+ 77"	- 185"
10	+ '3476	- 253	+ 5955	+ '0986	+ 94	- 121	+ '0743	+ 51	+ 959	- '0382	+ 77	- 122
20	+ '3595	- 191	+ 6237	+ '0959	+ 161	- 113	+ '0670	+ 63	+ 742	- '0476	+ 75	- 7
30	+ '3616	- 124	+ 6306	+ '0852	+ 222	- 104	+ '0588	+ 72	+ 501	- '0567	+ 70	0
40	+ '3559	- 53	+ 6148	+ '0673	+ 275	- 85	+ '0498	+ 78	+ 246	- '0650	+ 62	+ 6
50	+ '3426	+ 19	+ 5775	+ '0426	+ 316	- 67	+ '0406	+ 81	- 14	- '0723	+ 52	+ 12
60	+ '3233	+ 90	+ 5224	+ '0133	+ 347	- 45	+ '0319	+ 82	- 271	- '0783	+ 40	+ 17
70	+ '2976	+ 156	+ 4492	- '0210	+ 367	- 24	+ '0238	+ 80	- 510	- '0828	+ 27	+ 22
80	+ '2681	+ 216	+ 3649	- '0573	+ 374	- 1	+ '0160	+ 75	- 729	- '0856	+ 13	+ 26
90	+ '2351	+ 268	+ 2704	- '0953	+ 368	+ 20	+ '0092	+ 68	- 924	- '0868	0	+ 30
100	+ '2001	+ 310	+ 1700	- '1331	+ 352	+ 41	+ '0033	+ 60	- 1092	- '0867	- 13	+ 32
110	+ '1638	+ 343	+ 656	- '1704	+ 326	+ 59	- '0014	+ 50	- 1224	- '0850	- 25	+ 34
120	+ '1271	+ 367	- 398	- '2057	+ 291	+ 77	- '0048	+ 39	- 1321	- '0820	- 37	+ 34
130	+ '0919	+ 380	- 1406	- '2379	+ 250	+ 98	- '0070	+ 27	- 1382	- '0779	- 48	+ 33
140	+ '0581	+ 382	- 2386	- '2667	+ 201	+ 105	- '0080	+ 14	- 1409	- '0726	- 57	+ 34
150	+ '0267	+ 374	- 3290	- '2913	+ 148	+ 116	- '0079	+ 1	- 1404	- '0666	- 65	+ 31
160	- '0016	+ 356	- 4106	- '3116	+ 91	+ 122	- '0066	- 11	- 1365	- '0600	- 71	+ 28
170	- '0261	+ 329	- 4815	- '3267	+ 32	+ 128	- '0040	- 23	- 1280	- '0529	- 75	+ 25
180	- '0462	+ 294	- 5396	- '3364	- 26	+ 129	- '0003	- 35	- 1179	- '0451	- 78	+ 21
190	- '0619	+ 252	- 5852	- '3409	- 84	+ 126	+ '0044	- 46	- 1042	- '0374	- 78	+ 16
200	- '0724	+ 204	- 6159	- '3400	- 141	+ 121	+ '0100	- 56	- 879	- '0298	- 77	+ 11
210	- '0774	+ 149	- 6308	- '3334	- 195	+ 113	+ '0165	- 64	- 692	- '0224	- 73	+ 5
220	- '0769	+ 89	- 6299	- '3213	- 244	+ 103	+ '0236	- 71	- 485	- '0152	- 68	0
230	- '0710	+ 27	- 6155	- '3046	- 287	+ 88	+ '0312	- 76	- 267	- '0085	- 61	- 5
240	- '0583	- 36	- 5776	- '2817	- 321	+ 70	+ '0394	- 80	- 29	- '0026	- 53	- 11
250	- '0401	- 99	- 5258	- '2545	- 349	+ 51	+ '0478	- 81	+ 207	+ '0023	- 43	- 17
260	- '0164	- 161	- 4582	- '2234	- 366	+ 31	+ '0558	- 80	+ 442	+ '0064	- 31	- 22
270	+ '0120	- 220	- 3765	- '1890	- 375	+ 9	+ '0638	- 76	+ 668	+ '0094	- 19	- 26
280	+ '0453	- 273	- 2817	- '1517	- 371	- 13	+ '0710	- 70	+ 877	+ '0110	- 5	- 29
290	+ '0819	- 319	- 1766	- '1131	- 356	- 36	+ '0773	- 61	+ 1052	+ '0110	+ 9	- 32
300	+ '1210	- 355	- 646	- '0740	- 329	- 57	+ '0827	- 51	+ 1211	+ '0097	+ 23	- 33
310	+ '1617	- 380	+ 523	- '0359	- 291	- 78	+ '0865	- 39	+ 1322	+ '0067	+ 37	- 33
320	+ '2023	- 392	+ 1691	+ '0003	- 242	- 94	+ '0891	- 25	+ 1391	+ '0023	+ 49	- 33
330	+ '2411	- 390	+ 2740	+ '0323	- 184	- 109	+ '0897	- 10	+ 1406	- '0036	+ 59	- 31
340	+ '2766	- 376	+ 3833	+ '0594	- 118	- 119	+ '0885	+ 6	+ 1370	- '0109	+ 68	- 28
350	+ '3074	- 347	+ 4722	+ '0800	- 49	- 124	+ '0855	+ 23	+ 1260	- '0196	+ 74	- 24

Constant part of $\int d\pi$ (to be added to tabular quantities) Jupiter - 112",
Saturn - 24", Total - 136".

Constant part of $\int d\zeta$ (to be added to tabular quantities) Jupiter + 2700",
Saturn + 765", Total + 3465".

As an example of the use of these tables we may deduce the values already obtained for the revolution 1835-1910 (*M.N.*, vol. lxvii. pp. 410, 411, 519, 520). For Jupiter the values of g' at $u=90^\circ$, $u=270^\circ$, are $296^\circ.44$, $322^\circ.24$, for Saturn $199^\circ.52$, $214^\circ.74$.

Hence we obtain—

	$\int dn$	$\int d\varpi$	$\int d\zeta$
$2\ 90^\circ$	+ "1068	- 343"	- 1056"
" 270°	+ '0074	- 229	- 97
$12\ 90^\circ$	+ '0097	- 55	- 887
" 270°	- '0190	- 71	+ 3
Sum $2\ 2$	+ '1142	- 572	- 1153
" 12	- '0093	- 126	- 884

Now $\log a$ for this revolution = 1.2480 , whence we have to multiply $\int dn$ by number whose $\log$ is $.0262$, $\int d\varpi$, $\int d\zeta$ by number whose $\log$ is $.0105$.

Performing the multiplication and adding the constants we obtain—

	$\int dn$	$\int d\varpi$	$\int d\zeta$
2	+ "1213	- 698"	+ 1519"
12	- '0099	- 153	- 141
Results previously obtained { 2	+ '1219	- 698	+ 1532
{ 12	- '0101	- 153	- 142

A more complete investigation of the average motion of the comet's perihelion has been made, using the results of all the revolutions for which the perturbations have now been computed. It appears that the perihelion has a direct motion in longitude at nearly the same rate as the node, so that the periods of the various planets for co-ordinates $x' y'$ may be taken the same as those given for z' , p. 112. The value of ϖ for 1910 should be $305^\circ.64$, not $303^\circ.64$ as printed on p. 112.

Photographs of Comet d 1907 (*Daniel*). By Dr. Max Wolf.
(Plate 6.)

Comet Daniel has been photographed at the Observatory, Königstuhl, Heidelberg, on the following nights, with the following instruments:—

Date.	1½-in.	6-in. (a)	6-in. (b)	16-in. (a)	16-in. (b)	28-in.	Total Plates.
1907							
July 21	...	...	...	I	I	...	2
22	I	I	I	I	I	...	5
Aug. 2	...	...	...	...	...	2	2
4	I	...	...	I	I	4	7
7	I	...	...	I	I	2	5
8	...	...	...	I	I	3	5
9	...	...	...	I	I	4	6
11	...	...	...	I	I	2	4
14	I	...	...	...	...	2	3
27	...	...	...	I	I	...	2
10 n.							41

Six of these plates were taken by Mr. Scheifele, 12 by Dr. Kopff, and 23 by myself; 9 are stereos.

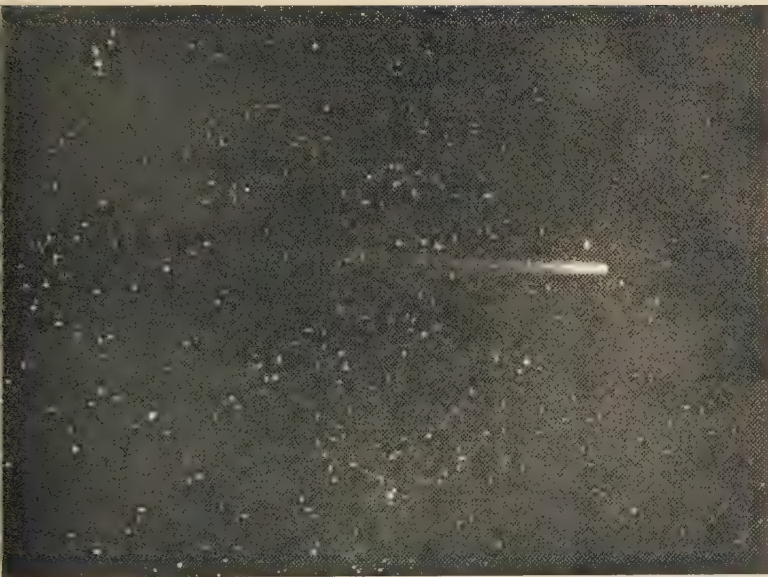
Besides these photographs I have made six drawings with the 28-in. reflector, viz. on August 4, 7, 8, 9, 11, and 14.

The most striking result appears to me the difference between the visual and the photograph appearances. All the drawings show a minimum of luminous radiation in or around the axis of the tail in the region turned away from the Sun. All the photographs show the brightest tails near the axis, exactly in the parts where visually there was a minimum of brightness. This very curious difference, which was remarked several years before in other comets photographed here, seems to be placed beyond doubt by the photographs taken with the reflector.

Besides this, it is very interesting, and perhaps in opposition with accepted theories, that the short arms of the tails furthest outside the axis are nearly all curved.

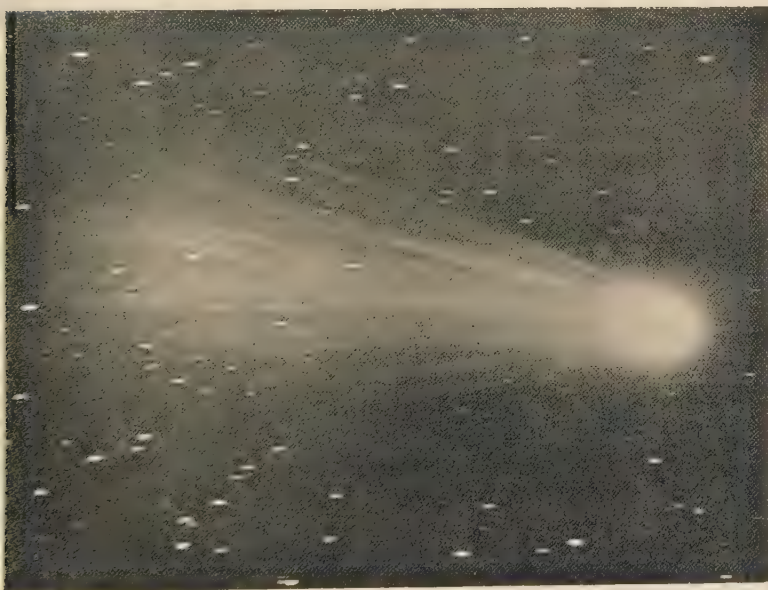
The following eight specimens are presented to the Society:—

			h	m			m
1.	Plate D144,	Reflector,	1907 Aug.	2 ; 14	44'6	Königstuhl M.T., exposure	10
2.	" D147,	"	"	4 ; 14	35'1	"	15
3.	" D152,	"	"	7 ; 14	52'1	"	15
4.	" D155,	"	"	8 ; 14	42'2	"	25
5.	" D161,	"	"	9 ; 14	59'6	"	12
6.	" D167,	"	"	11 ; 15	5'2	"	15
7.	" D173,	"	"	14 ; 15	8'2	"	12
8.	" A4850,	1½-in. lens,	"	14 ; 15	0'2	"	20



1907 Aug. 14 d. 15 h. 0'2 m.

1 1/2-IN. LENS.



1907 Aug. 14 d. 15 h. 8'2 m.

28-IN. REFLECTOR.

COMET *d* 1907 (DANIEL). MAX WOLF, HEIDELBERG.

The lantern slides are a little enlarged from the original negatives. On the slides from the reflector-plates 1 degree equals 86 mm., and on the slide of the $1\frac{1}{4}$ -in. lens 1 degree equals 4 mm.*

It is interesting to compare photographs 7 and 8, which were taken at the same time, but with very different optical means—28-in. and $1\frac{1}{4}$ -in.

The tail on No. 8 is somewhat over 12 degrees long, while the small field of the reflector gives the head and its neighbourhood only. The bright star near the head of the comet on No. 8 is γ Geminorum.

* In the reproduction on Plate 6 one degree on the reflector plate = 79 mm., and one degree on the $1\frac{1}{4}$ -in. lens plate = 3.7 mm.

*Astrophysical Observatory,
Heidelberg (Königstuhl),
1907 December 10.*

On an improved method of illuminating the field in a Transit Instrument, and its effect on the discordance in reversed positions of the instrument. By Sir W. H. M. Christie, K.C.B., F.R.S., and H. A. H. Christie, B.A. (Plates 7, 8.)

It is known that with a reversible transit instrument observations made in the two positions of the instrument generally show discordant results for clock error for stars reduced with the same collimation error for the two positions. This discordance has been attributed to what is called lateral flexure, and a correction depending on $\cos Z.D.$ has sometimes been applied, but at Greenwich we never felt satisfied that there was any real ground for this assumption. We have suspected sometimes that it was due to looseness in the mounting of the object-glass, and in the Paris-Greenwich longitude determination of 1888 this was found to be the case with both the portable transits. But though this was remedied by an improvement in the mounting of the object-glass, this puzzling discordance still remained in subsequent longitude determinations. Whilst there is in each series of observations a general tendency in one direction, there are considerable variations in the discordance between the mean clock errors for each night in the two positions of the instrument, as will be seen from Tables I., II., III., IV., V., VII., and VIII.

Mr. Hollis, on thinking over the matter in connection with the observations for longitude of Killorglin in 1898 and of Paris in 1902, had come to the conclusion that to obtain the correct clock-error from the mean of results in reversed positions it was necessary to keep the focus of the eyepiece absolutely fixed, and that to secure this it was expedient to keep the diagonal eye-tube which was used for the observations, and was liable to sag, rigidly attached to the tube of the instrument. In the Paris longitude

observations in 1902 special attention was paid to this, the eyepiece being adjusted to focus at the beginning of the evening, and not altered afterwards. The discordance, however, was not altogether got rid of, and on some nights it exceeded $0^s.2$ or $0^s.3$. There was the further difficulty that, as will presently be explained, the observer's eye is liable to change of focus when the bright star comes into the field.

In 1906 and 1907 Mr. Harold Christie was working with two of the portable transits (B and C) previously used in the Paris longitude determinations of 1888 and 1902, with a view to other longitude determinations, and the large discordances in his results for the two positions again called attention to the question. He noticed that if the eyepiece was adjusted for distinct vision of the wires in an illuminated field a fresh adjustment was required when the star appeared, the eyepiece having to be pulled out further from the object-glass owing to change in the focus of the eye, the wires being, by an unconscious mental process, naturally referred to a finite distance (say 2 or 3 feet), and the star, when it came in, to an infinite distance. This change of focus still took place when the other eye was covered up. He further noticed (on March 1, 1907) that while adjusting the eyepiece for focus on a slow-moving polar star the star seemed to cross the wire from one side to the other as the eyepiece was moved in or out, the apparent movement of the wire being as much as $20''$. A similar effect had been previously noticed on the meridian mark (a bright point) seen in an illuminated field. It was at first thought that this might be due to the wires not being exactly in the focus of the object-glass, but readjustment of the focus failed to get rid of the effect. On trying the same experiment on Polaris by daylight next day it was found that no apparent movement of the wires could be got by movement of the eyepiece in or out. This indicated that the effect was due to *illumination of the field*. It was further noted that the movement was less apparent with a strong illumination than with a reduced light. It may here be explained that the illumination of the field is given by a gilt annular reflector in the transit axis, the inclination of which can be varied from a minimum of 45° to a greater angle with the axis so as to reduce the light.

Afterwards observations of stars were made with the eyepiece inside the focus, at the best focus, and outside focus. The results were that the discordance $W-E$ was positive and large ($+1^s.5$) with the eyepiece far in, and negative (about $-0^s.6$ or $-0^s.7$) with the eyepiece far out.

It is to be observed that the mean of the results W and E is sensibly unaffected by the discordance, as will be seen from the last column.

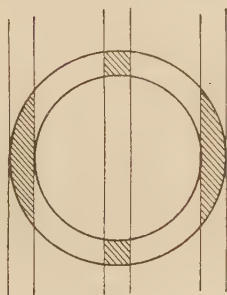
Transit C.—Annular Illumination Experiments with Eyepiece focus.
By Mr. H. Christie.

Date.	Position of Eyepiece.	No. of Stars. W. E.	W—E. S	Mean. Clock slow. S
1907.				
March 19	Good focus	2, 2	+0'15	+59'48
	Inside focus	1, 2	+1'45	59'66
	Outside focus	2, 2	-0'73	59'45
March 20	Good focus	3, 2	+0'58	59'92
	Inside focus	2, 2	+1'40	59'91
	Outside focus	3, 3	-0'55	59'96

N.B.—The “good focus” position was the ordinary one for star observations; the eyepiece was displaced from this position until observation was only just possible.

These experiments seemed to clearly establish the connection between the discordance and eyepiece focus, and they further suggested that the annular reflector was in fault.

It is to be noted that the wires at night are not seen by their own light, but as dark shadows cutting off the light of the illuminated field, and thus the illuminating surface, which casts the shadow of the wire, has an important influence on its appearance with the annular reflector. When the wire is out of focus the



shadow of each point of the wire is an annulus, and the superposition of these will give two dark lines parallel to the wire with a comparatively slight haze between, the two sides of the annulus being more effective in forming the shadow images than the top and bottom, as will be seen from the accompanying figure, which shows the portions of the annulus at the sides and at the top and bottom respectively, which are effective in forming the corresponding shadow images of a vertical wire. Thus two fairly clear and separated images are formed one on each side of the true position of the wire.* Now if one side of the annulus is more distinctly

* On examination of the wires by daylight illumination from the object-glass, it was found that when the eyepiece was out of focus, instead of two images a single image was seen, with bands on each side.

lighted than the other, and the whole illumination of the field is not very bright, the eye will lose one of the two images of the wire, and only see a single image, which will appear fairly clear, and will be displaced by an amount depending on the diameter of the out-of-focus annular ring.

On investigation this proved to be the case. When the annular reflector was at an angle of 45° to the axis (giving a maximum of illumination) it was fairly evenly illuminated, but when the inclination was increased, in order to reduce the light to a convenient amount for star-observing, one side of the gilt reflector became considerably brighter than the other. This was the actual position always used in practice, as the full illumination was too strong for observing stars. Also, it was found that the side of the reflector which gave most light was the one which would, by the above theory, cause the clock error micr. W to be greater than the clock error micr. E, as the star-observations showed to be the case. In order further to test this, Mr. H. Christie made observations on April 5, 6, and 10 with the reflector kept at 45° to the axis (a rheostat being used to reduce the light from the electric lamp), and obtained better results.

It had long been felt that illumination by means of an annular reflector in the transit axis was not satisfactory, as the image formed at the eye-ring from the illumination of the field was an annulus outside the circle formed by a star, and there was risk of the eye failing to receive the whole of the rays on both sides coming from the reflector.

Illumination by rays coming centrally within the cone of rays from the object-glass would be preferable in this respect, but there are objections to the way in which this has usually been carried out. In 1870, Sir G. B. Airy successfully applied central illumination to the Water Telescope which he planned, mounting a piece of looking-glass at an angle of 45° in front of the object-glass to receive the light from a gas flame, without any condensing lens. In this way the light was well diffused over the field, the gas flame as well as the mirror being well out of focus at the plane of the wires.

But in other forms which have been used since, a condensing lens with a small electric lamp in its focus has been introduced, which, in combination with a small reflecting prism or other specular reflector fixed in front of the object-glass, forms a magnified image of the source of light in the plane of the wires, exaggerating its defects, and making it difficult to get tolerable uniformity of illumination of the field and sharp definition of the wires.

In planning the New Altazimuth, a sketch arrangement was proposed in 1896 January for central illumination by light reflected from the axis lamp inside the telescope tube to a matt surface gilt reflector attached centrally to the inside surface of the object-glass; but, owing to practical difficulties, this plan was not carried out, and the ordinary annular reflector in the axis was at first

used. In 1897 September central illuminations by the usual method of small reflecting prism ($\frac{3}{8}$ -inch square) cemented on the outside of the object-glass and collimating lens with a small electric lamp in its focus was arranged for, and was subsequently brought into use. There was difficulty, however, with the illumination, owing to the filaments of the lamp forming an enlarged image in the focus, and the wires were badly defined, owing, as afterwards appeared, to diffraction effects from the small aperture of the illuminating pencil. Further, with such a small pencil, dust on the field lens of the eyepiece would be sensibly in focus with the wires.

Attention being again called to the question, it seemed that too much importance had been given to simply getting enough light without sufficient consideration of the optical conditions required in the arrangement for central illumination of the field. These conditions are:—

- (1) That a uniform illumination shall be secured over the field, without such defects as are due to the source of light coming to a focus near the plane of the wires.
- (2) That the illuminating pencil should have a sufficient aperture to minimise diffraction effects on the wires, giving rise to shadow-bands.

The condition (1) can be secured by substituting a matt surface, such as opal glass (finely ground) or plaster of Paris, for the specular reflector, the condensing lens with electric lamp in its focus being retained. The parallel rays which fall on the matt surface from each point of the source of light in the focus of the condensing lens are scattered uniformly in directions slightly inclined to the axis of the telescope, and after passing through the object-glass, converge respectively to each point of the field, so that each point is illuminated by a pencil uniformly distributed over the whole of the opal reflecting surface.

As regards condition (2), it is to be borne in mind that in transit observations sharp definition of the wires is as important as sharp definition of stars; and that, as the wires are seen as shadows thrown by the illuminated field, the aperture of the illuminating pencil determines the sharpness of the wires. With the small pencils which have commonly been used even on large instruments, there are necessarily large diffraction effects which are serious with the relatively very high powers employed. At the same time it is to be noted that comparatively little loss of light, and practically no loss of definition for a star, is entailed by cutting out a circle of, say, one-fourth of the aperture at the centre of the object-glass.*

After some preliminary experiments with central illumination for the portable transit, the following arrangement was adopted on 1907 May 8 to secure the two conditions stated above. An elliptical finely ground opal glass was used as the reflector, and mounted in a brass tube which was attached to the outer surface of the object-glass by shellac. The illuminating source was a small

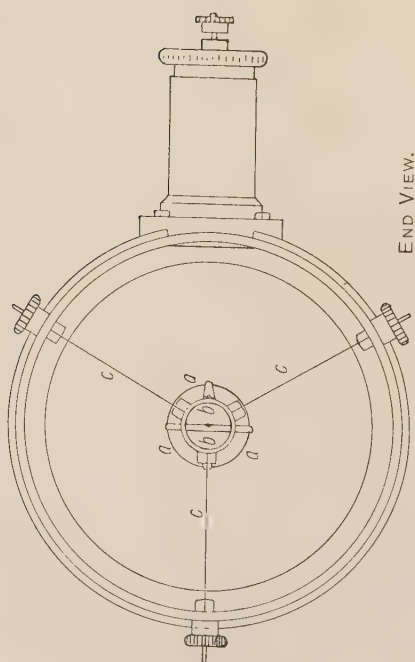
* The loss of light would in this case only be one-sixteenth.

electric lamp in the focus of a condensing lens, throwing a parallel beam of light on the opal reflector. The brightness of the electric lamp was controlled by a rheostat, and a very satisfactory illumination of the field, with the wires sharply defined, was obtained. The diameter of the pencil of rays from the reflector was $\frac{3}{4}$ -inch, which was adopted as suitable for the 3-inch object-glass. The results given for Transit C, in Table IX.—central illumination—were all obtained with this arrangement. It was found, however, that the shellac was not a secure fastening for such an instrument as a portable transit, as any jar was apt to dislodge the arrangement, so a more secure form of support was adopted later on, and applied first to transit B in July 1907. This consisted of an opal reflector as before, fixed to the dew cap by three pieces of watch-spring, and is shown in Plate 7. This arrangement was found satisfactory, and so was attached to the other instruments (transits B and D) as well. No trouble has been caused by diffraction effects. The observations obtained are, on the whole, satisfactory, although two nights (September 10 and 11) with transit B give discordances W—E of over $0^{\circ}.1$. There was probably some other cause at work on these nights, as the collimation determined from the nadir gave a sudden jump of about $2''$ in the *opposite* direction to the error shown by the stars (Tables VI., IX., and X.).

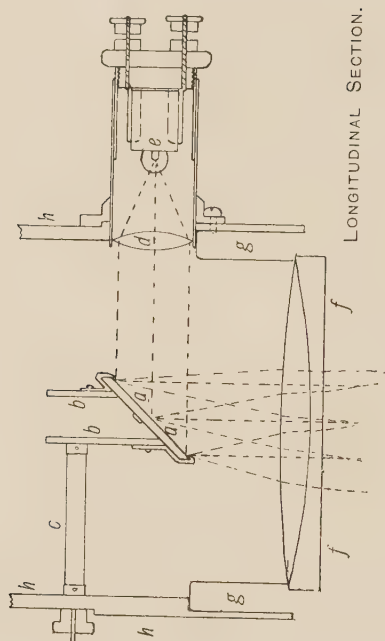
As the application of the improved central illumination to the small transits proved so successful, it was applied to the New Altazimuth on 1907 June 5, the mode of attaching the reflector to the object-glass being slightly modified from the original plan with shellac cement. In order to secure a firmer attachment, a plain glass plate, worked slightly concave on the outside to fit the curvature of the outer surface of the object-glass, was burnished into the end of the tube carrying the opal reflector, and cemented on to the object-glass with Canada balsam. In this way the obstruction of light due to the attachment is reduced to a minimum, viz. the thickness of the brass tube. The general arrangement is shown in Plate 8. The aperture of the illuminating pencil adopted for the 8-inch object-glass was $1\frac{1}{2}$ inches, which was found to give greatly improved definition of the wires, which had been very unsatisfactory with the $\frac{5}{8}$ -inch pencil formerly in use. At the same time the troublesome markings previously seen in the illuminated field were got rid of, and a satisfactory uniform illumination substituted.

It is proposed to apply a similar method of illumination to the Greenwich transit-circle as soon as the necessary electrical connections can be made.

The tables show the difference between the mean clock error determined with the instrument micrometer West and micrometer East respectively. They are corrected for clock rate and reduced with a collimation error determined from nadir observations only. The weights given depend on the number of reversals of the instrument, a weight of 1 corresponding to a single reversal, while a weight of 2 corresponds to two or more.



END VIEW.

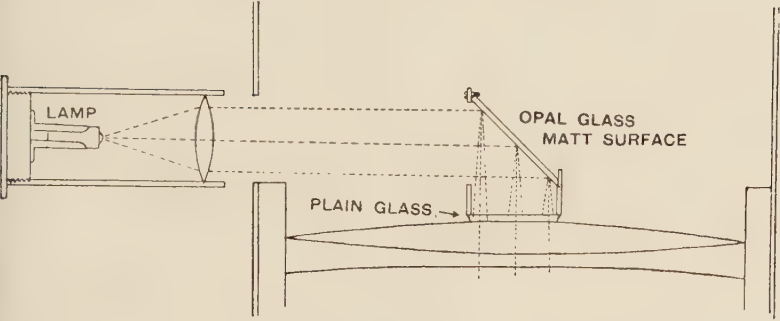


LONGITUDINAL SECTION.

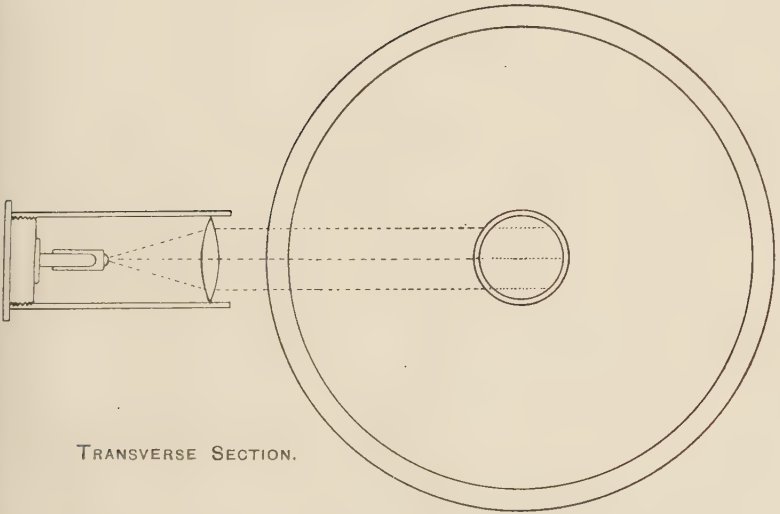
PORTABLE TRANSIT—ILLUMINATION OF FIELD.

- a*, Matt surface reflector (Opal glass finely ground).
b, Brass tube to carry reflector.
c, Supports for tube, made of watch-spring.
d, Condensing lens.
e, Small electric lamp.
f, Object-glass of telescope.
g, Object-glass cell.
h, Dew cap.

ALTAZIMUTH-ILLUMINATION OF FIELD.



LONGITUDINAL SECTION.



TRANSVERSE SECTION.

TABLE I.

Transit B.—Trial Observations by Mr. Dyson and Mr. Hollis.
Annular Illumination.

Date. 1898.	W—E. Means.	Observer.
s		
June 25	+0.16	H
July 2	+0.26	H
8	+0.14	H
16	+0.25	H
Aug. 2	+0.39	H
Sept. 3	+0.30	H
21*	+0.82	H
23*	+0.69	H
23	+0.16	H
23	+0.19	D
26	+0.35	H
26	+0.37	D
Mean	<u>+0.340</u>	

TABLE II.

Date. 1901.	W—E. Means.	No. of Stars. W. E.	Obsr.
s			
Nov. 23	+0.37	7, 7	H
Dec. 2	+0.40	4, 5	H
4	-0.02	4, 5	H
10	+0.20	7, 6	H
10	+0.25	4, 5	H
16	+0.34	6, 5	H
16	+0.41	5, 4	H
1902.			
Jan. 25	+0.32	5, 5	H
28	+0.38	4, 6	H
Feb. 10	+0.49	5, 5	D
11	+0.08	6, 6	D
Mean	<u>+0.293</u>		

On Sept. 20 the object-glass was taken out, examined, and replaced.

* Ramsden eyepiece with prism used for these observations instead of diagonal eyepiece.

On Dec. 10 the position of the illuminating lamp was altered and purposely made unsymmetrical, an apparently bad arrangement.

TABLE III.

Transit B.—Observer, Mr. Dyson.
Annular Illumination.

Date. 1902.	W—E. Means.	No. of Stars. W. E.	Wt.
s			
Paris.			
Mar. 18	-0.143	6, 5	1
19	-0.065	19, 12	2
23	+0.002	11, 17	2
25	+0.140	12, 19	2
28	-0.047	12, 13	1
Mean	<u>-0.005</u>		
Date. 1902.	W—E. Means.	No. of Stars. W. E.	Wt.
Greenwich.			
Apr. 6	-0.036	5, 5	1
8	+0.046	14, 15	2
10	+0.137	10, 8	1
12	+0.066	11, 9	1
13	+0.107	18, 21	2
17	-0.005	18, 16	2
18	+0.076	18, 18	2
20	+0.034	7, 8	1
22	+0.030	15, 16	2
23	+0.108	22, 21	2
24	+0.134	21, 18	2
Mean	<u>+0.066</u>		

TABLE III.—*continued.*

Date. 1902.	W—E. Means.	No. of Stars. W. E.	Wt.	Date. 1902.	W—E. Means.	No. of Stars. W. E.	Wt.
Paris.				Paris.			
Apr. 28	+0.088	13, 20	2	Oct. 6	+0.068	10, 7	1
May 1	+0.059	20, 17	2	7	+0.127	12, 16	2
3	+0.024	18, 17	2	8	+0.029	6, 10	1
Mean	<u>+0.057</u>			9	-0.006	9, 9	1
				10	+0.047	17, 17	2
				13	+0.113	12, 7	1
				14	+0.080	20, 20	2
				16	+0.076	15, 17	2
				17	+0.073	21, 19	2
				21	+0.051	19, 15	2
				22	+0.024	16, 15	2
				Mean	<u>+0.064</u>		
Greenwich.				Greenwich.			
Sept. 21	+0.015	3, 2	1	Oct. 26	+0.149	9, 2	1
22	+0.099	18, 19	2	27	+0.212	11, 11	1
23	-0.004	8, 5	1	29	+0.048	18, 19	2
24	+0.158	19, 18	2	30	+0.005	21, 10	1
25	+0.087	8, 10	1	Nov. 1	+0.107	9, 8	1
26	+0.127	17, 16	2	2	+0.040	9, 8	1
Mean	<u>+0.096</u>			3	+0.114	6, 9	1
				4	+0.101	24, 23	2
				Mean	<u>+0.093</u>		

TABLE IV.

*Transit B.—Observations by Mr. Storey and Mr. Witchell.**Annular Illumination.*

Date. 1906.	W—E. Means.	No. of Stars. W. E.	Obsr.	Date. 1906.	W—E. Means.	No. of Stars. W. E.	Obsr.
s				s			
Jan. 22	+0.07	6, 6	J. S.	Jan. 30	+0.15	4, 4	J. S.
23	+0.06	4, 2	W.	Feb. 3	+0.03	4, 4	J. S.
24	+0.13	7, 6	J. S.	April 4	+0.04	4, 4	J. S.
26	+0.09	7, 6	J. S.	9	+0.17	4, 3	W.
29	+0.15	4, 4	W.	19	+0.12	5, 5	W.
				Mean	<u>+0.101</u>		

On some other nights individual stars are very discordant; in one case there is a range of one second of time.

TABLE V.

TABLE VI.

Transit B.—Observer, Mr. H. Christie.

<i>Annular Illumination.</i>				<i>Central Illumination.</i>			
Date. 1906.	W—E. Means.	No. of Stars. W. E.	Wt.	Date. 1907.	W—E. Means.	No. of Stars. W. E.	Wt.
July 16	+0°081 ^s	2, 6	1	July 30	+0°053 ^s	3, 4	1
18	+0°369	4, 5	2	Sept. 7	+0°091	7, 8	2
20	+0°868	5, 7	1	9	+0°089	8, 8	2
23	+0°480	3, 6	1	10*	+0°109	8, 7	2
24	+0°584	7, 7	1	11*	+0°131	7, 7	2
25	+0°972	15, 7	2	15	-0°030	4, 4	1
31	+0°368	8, 6	1	19	+0°038	5, 6	2
Aug. 7	+0°703	7, 4	1	24	-0°023	9, 12	2
Sept. 20	+0°093	3, 7	1	25	-0°052	7, 7	2
25	+0°744	5, 7	2	26	-0°034	15, 15	2
28	+0°521	4, 2	1	Oct. 4	+0°004	8, 8	2
Oct. 9	+1°057	3, 9	1	Mean	+0°036		
11	+0°409	2, 2	1				
22	+0°725	2, 3	1				
23	+0°088	2, 3	1				
27	+0°455	4, 3	1				
Mean	+0°558						

Observer, Captain Monroe, R.N.

Sept. 16	+0°042	4, 3	1
17	-0°052	6, 6	2

Note.—The observations in Table VI. (Central Illumination) were all reduced with the same collimation error throughout; there appears to have been a slow change of collimation, which was also shown by the nadir observations. No correction has been applied for inequality of the pivots. The mean of a number of determinations made by the striding level in 1902, when the pivots were last reground, was 0''00.

* On September 10 and 11 the nadir readings for collimation are very discordant, differing by about 1'' from the mean. The clock stars appear to be affected also, but with the opposite sign.

TABLE VII.

*Transit C.—Observer, Mr. Hollis.**Annular Illumination.*

Date. 1902.	W—E. Means.	No. of Stars. W. E.	Wt.	Date. 1902.	W—E. Means.	No. of Stars. W. E.	Wt.
Greenwich.				Paris.			
Mar. 17	+0'128	15, 12	2	Sept. 21	+0'204	15, 11	2
19	+0'106	17, 17	2	22	+0'120	16, 16	2
20	+0'056	10, 12	1	24	+0'121	17, 14	2
21	+0'086	10, 7	1	26	+0'118	16, 18	2
22	+0'066	14, 11	1	Mean	+0'141		
23	+0'111	8, 8	1				
25	+0'260	16, 18	2	Greenwich.			
27	+0'416	13, 10	1	Sept. 29	+0'133	14, 14	2
28	+0'070	14, 5	2	Oct. 2	+0'032	16, 16	2
Mean	+0'143			8	+0'079	20, 19	2
Paris.				9	+0'077	10, 7	1
Apr. 7	+0'303	10, 8	1	10	-0'019	19, 20	2
8	+0'316	9, 2	1	12	+0'057	12, 8	1
9	+0'067	10, 15	1	14	+0'005	19, 20	2
11	+0'095	9, 11	1	15	+0'087	14, 10	1
13	+0'267	16, 18	2	16	+0'081	20, 18	2
17	+0'217	17, 16	2	17	+0'033	16, 17	2
18	+0'221	17, 16	2	18	+0'138	17, 14	2
20	+0'162	18, 11	2	21	+0'041	20, 20	2
21	+0'166	16, 16	2	22	+0'199	15, 18	2
23	+0'260	9, 15	1	Mean	+0'072		
24	+0'137	17, 19	2				
Mean	+0'199			Paris.			
Greenwich.				Oct. 25	-0'003	14, 13	2
Apr. 27	+0'052	9, 14	1	27	+0'058	10, 8	1
28	+0'060	18, 17	2	28	+0'061	7, 13	1
May 1	+0'064	16, 12	2	29	+0'158	16, 17	2
2	+0'070	10, 14	1	31	+0'172	16, 17	2
3	+0'007	17, 17	2	Nov. 2	+0'157	17, 11	2
Mean	+0'048			3	+0'187	20, 17	2
				4	+0'234	20, 19	2
				Mean	+0'138		

TABLE VIII.

TABLE IX.

Transit C.—Observer, Mr. H. Christie.

Annular Illumination.

Central Illumination.

Date. 1906.	W—E. Means.	No. of Stars. W. E.	Wt.
s			
Nov. 5	−0°015	7, 7	1
11	+0°652	2, 7	1
19	−0°281	6, 4	1
22	+0°118	14, 8	2
Dec. 9	+0°253	3, 3	1
10	+0°316	4, 5	1
13	+0°507	11, 10	2
14	+0°410	3, 3	1
1907.			
Jan. 21	+0°386	7, 9	1
26	+0°436	7, 7	2
29	+0°321	5, 3	1
Feb. 1	+0°105	8, 6	1
22	+0°323	17, 13	2
28	+0°406	15, 17	2
Mar. 1	+0°146	9, 8	2
10	+0°010	2, 2	1
11	+0°435	4, 8	2
22	+0°185	4, 6	2
26	+0°198	3, 5	1
Mean	<u>+0°277</u>		

Date. 1907.	W—E. Means.	No. of Stars. W. E.	Wt.
s			
May 8	−0°029	16, 15	2
10	+0°041	1, 6	1
27	−0°015	7, 5	2
June 2	−0°023	12, 11	2
10*	−0°001	11, 5	2
17	−0°045	5, 6	2
19	+0°046	15, 14	2
July 16	+0°047	7, 7	2
17	+0°055	2, 2	1
Mean	<u>+0°003</u>		

* Observer, Mr. Hollis.

Note.—No correction for inequality of the pivots has been applied. The following determinations of the apparent inequality have been made with the striding level, showing that it is very small.

Date.	Apparent Inequality of Pivots.	Discordance W—E due to Pivot Error.
s		
1902 March–May	+0°05	+0°016
1902 Sept. –Nov.	−0°09	−0°029
1906 Nov. –Dec.	−0°09	−0°022
1907 Jan. –March	−0°12	−0°029
1907 April –June	−0°07	−0°017

The pivots have not been re-ground since 1901.

TABLE X.

Transit D.—Central Illumination.

<i>Observer, Captain Monro, R.N.</i>				<i>Observer, Lieut. Gibson, R.N.</i>			
Date. 1907.	W—E. Means.	No. of Stars. W. E.	Wt.	Date. 1907.	W—E. Means.	No. of Stars. W. E.	Wt.
	s				s		
Sept. 18	-0'137	2, 3	1	Sept. 27	-0'149	3, 3	1
19	+0'085	8, 10	2	Oct. 2	+0'019	7, 7	2
23	+0'030	10, 8	2	4	+0'096	8, 8	2
24	+0'009	12, 10	2	7	-0'089	6, 4	1
30	+0'008	5, 8	2	8	-0'020	5, 8	2
Oct. 2	-0'014	8, 6	2	Mean	-0'006		
4	+0'150	9, 9	2				
7	-0'052	8, 6	2				
8	+0'021	8, 8	2				
Mean	+0'020						

On the Orbit of the Binary Star β 80.

By T. J. J. See, A.M., Ph.D. (Berol.).

This interesting and rapidly revolving system was discovered by Burnham with his celebrated 6-inch telescope at Chicago in 1874. It was measured by Dembowski the following year, and has since been followed regularly by the most active observers of close pairs. At first the motion was very slight, because the companion was near apastron, with a long radius vector and revolving slowly. But in the last ten years the motion has become very rapid both in angle and distance. The apparent orbit quite well represents the observations, as shown by the accompanying diagram. The material used is the complete measures given by Burnham in his General Catalogue of Double Stars, just published by the Carnegie Institution. The place of β 80 for the epoch of 1880 is $\alpha = 23^{\text{h}} 12^{\text{m}} 45^{\text{s}}$; $\delta = +4^{\circ} 45'$; mags. 8.2, 9.1.

The elements are as follows:—

$$\begin{aligned}
 P &= 63.5 \text{ years} & \Omega &= 107^{\circ}.8 \\
 T &= 1905.30 & i &= 17^{\circ}.6 \\
 e &= 0.726 & \lambda &= 10^{\circ}.7 \\
 a &= 0''.626
 \end{aligned}$$

Apparent orbit:—

Length of major axis	= 1".25
Length of minor axis	= 0".79
Angle of major axis	= 117°.4
Angle of periastron	= 117°.8
Distance of star from centre	= 0".452

No previous orbit for this star has been worked out. Some astronomers have expressed the opinion that the period could not be deduced until the companion returned to apparent apastron. But it will be seen that the observations fix the apastron with singular accuracy.

Comparison of computed with observed places for β 80.

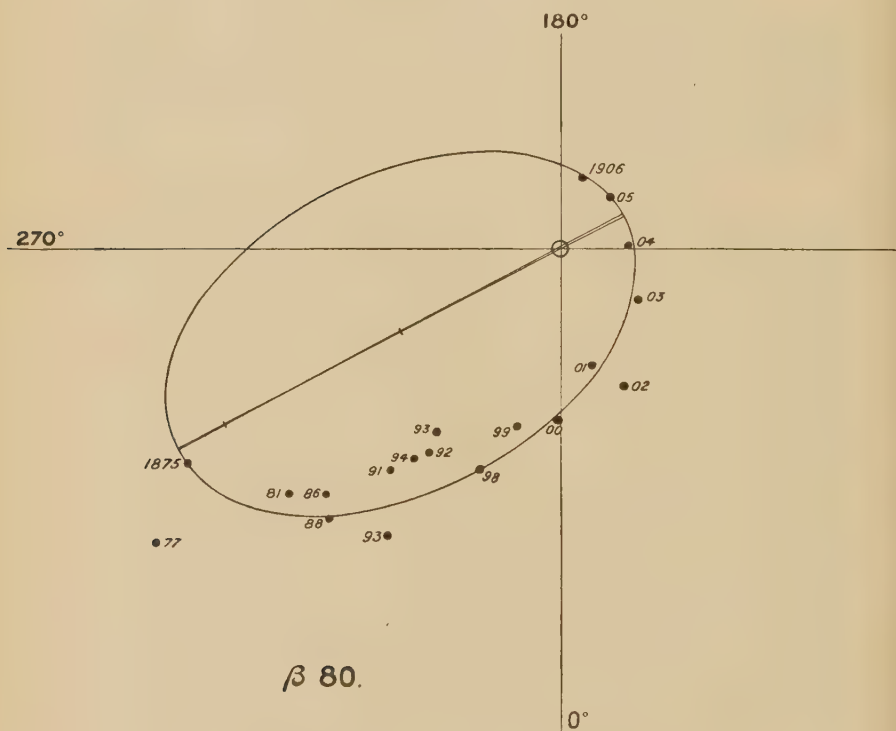
t	θ_0	θ_c	$\theta_0 - \theta_c$	ρ_0	ρ_c	$\rho_0 - \rho_c$	Observer, Nights.
1875.80	300°.4	300°.9	-0°.5	1".07	1".07	0".00	Δ , 5.
1877.79	306°.1	302°.4	+3°.7	1".24	1".07	+0".17	β , 2.
1881.69	312°.2	308°.6	+3°.6	0".91	1".02	-0".11	β , 3.
1886.94	316°.1	316°.7	-0°.6	0".84	0".92	-0".08	LM, 1.
1888.75	319°.5	319°.8	-0°.3	0".88	0".87	+0".01	β , 4; Lv, 3.
1891.57	322°.5	325°.5	-3°.0	0".69	0".78	-0".09	β , 3.
1892.97	327°.2	329°.0	-1°.8	0".60	0".74	-0".14	Sp, 3.
1893.60	328°.5	330°.5	-2°.0	0".83	0".71	+0".12	Lv, 1; W, 3.
1894.84	325°.1	334°.4	-9°.3	0".63	0".66	-0".03	Lewis, 1.
1898.77	340°.3	336°.0	+4°.3	0".59	0".64	-0".05	β , 1; Bow, 1; A, 4.
1899.38	346°.0	338°.7	+7°.3	0".45	0".61	-0".16	Bry, 4.
1900.65	358°.6	351°.6	+7°.0	0".42	0".47	-0".05	Doo, 2; A, 2.
1901.83	13°.8	12°.4	+1°.4	0".30	0".33	-0".03	Bry, 4; Doo, 2.
1902.75	24°.3	29°.8	-5°.5	0".37	0".26	+0".11	Bry, 1; Lewis, 1.
1903.76	55°.3	53°.4	+1°.9	0".22	0".21	+0".01	A, 2; Bry, 2.
1904.73	91°.6	90°.2	+1°.4	0".16	0".18	-0".02	A, 2.
1905.65	136°.4	134°.8	+1°.6	0".17	0".17	0".00	A, 2.
1906.52	163°.8	172°.4	-8°.6	0".18	0".19	-0".01	A, 2.

Ephemeris.

t	θ_c	ρ_c	t	θ_c	ρ
1908.7	209°.0	0".28	1914.7	256°.4	0".60
1909.7	222°.5	0".33	1915.7	260°.2	0".65
1910.7	233°.3	0".39	1916.7	263°.0	0".69
1911.7	241°.3	0".45	1917.7	265°.7	0".72
1912.7	247°.2	0".50	1918.7	268°.2	0".76
1913.7	252°.4	0".56			

Dembowski measured the position here laid down on five nights in 1875. It happens that his observations both in angle and in distance are perfectly represented by this ellipse which satisfies the law of areas. Thus both ends and a full side of

the apparent ellipse have already been described, and the observations of Aitken in 1905 and 1906, beyond periastron, show that the other side of the ellipse is almost identical with that already described. The elements are thus fixed with unusual accuracy. It seems likely that the period is not uncertain by more than one or two years, and that the eccentricity cannot depart from the value here given by more than ± 0.03 .



This orbit illustrates the value of good observations advantageously placed. An ephemeris for the next ten years is added for the convenience of observers. This system deserves careful attention for some time, because the motion will be rapid both in angle and distance, and observations will be valuable in verifying the orbit.

As Burnham's General Catalogue gives all the measures used, it suffices to give a calculated comparison with the annual means as plotted in the diagram, which embraces all the best material.

U.S. Naval Observatory,
Mare Island, California,
1907 December 4.

On the Orbit of β 513 = 48 Cassiopeiae.

By T. J. J. See, A.M., Ph.D. (Berol.).

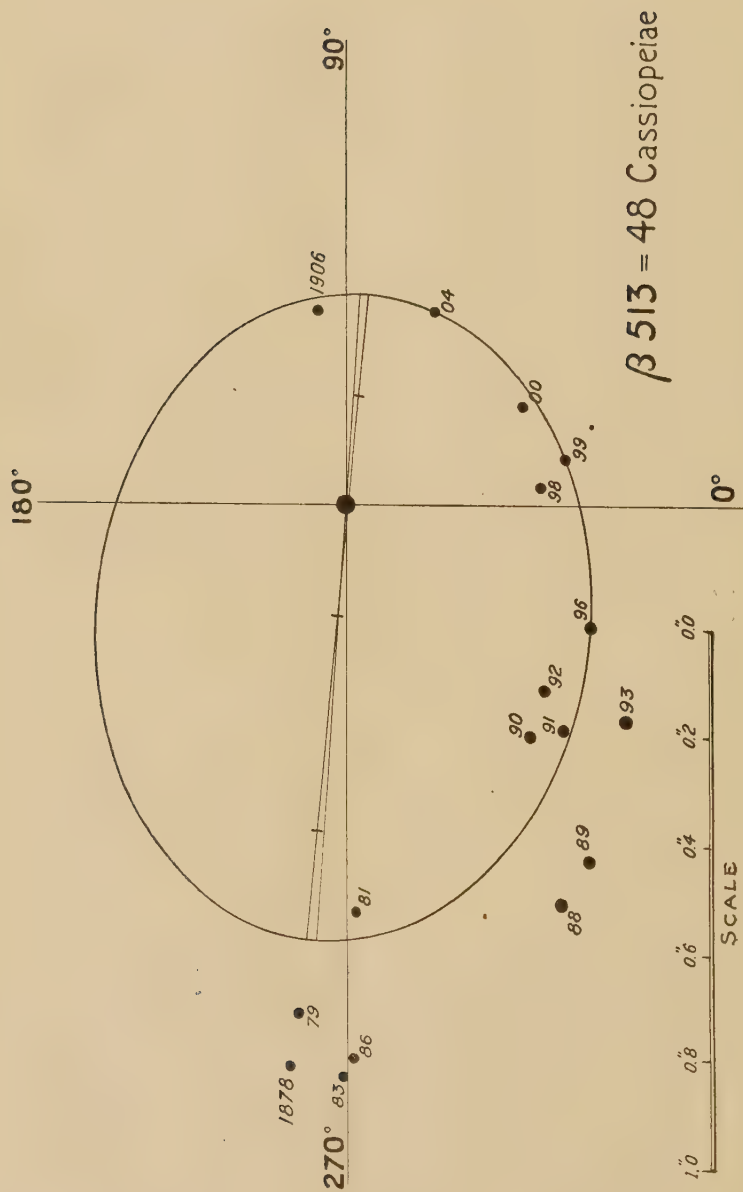
This remarkable binary was discovered by Burnham with the 18½-inch refractor at Chicago in 1878. Its place for the epoch 1880 is $\alpha = 1^h 52^m 7^s$; $\delta = +70^\circ 19'$; and the magnitudes of the components are 5.0 and 7.0. The star is always close, and presents the same difficulty in dividing the components as the systems 99 Herculis, 85 Pegasi, and τ Cygni, which can be well separated only by powerful instruments in good seeing. The measures of such unequal pairs are often vitiated by sensible systematic errors, so that the computer encounters difficulty in reconciling the observations to any orbit.

The orbit herewith presented gives the general character of the motion with essential accuracy, and will be useful to observers for the next ten or fifteen years, while the companion is again receding towards apastron.

The two ends and one side of the apparent ellipse have been described. The principle of areas shows that the orbit has a rather small eccentricity, and that it is not highly inclined. I am indebted to Professor Eric Doolittle, of the Flower Observatory of the University of Pennsylvania, for the last measure, which falls exactly on the orbit here laid down. His note on the measure was, "I am certain this is real; a perfect night."

Comparison of computed with observed places for β 513.

t	θ_0	θ_c	$\theta_0 - \theta_c$	ρ_0	ρ_c	$\rho_0 - \rho_c$	Observer, Nights.
1878.70	264.5	265.1	- 0.6	1.05	0.82	+ 0.23	Δ , 1; β , 3.
1879.40	265.0	267.2	- 2.2	0.95	0.82	+ 0.13	$O\Sigma$, 1; β , 2.
1881.67	271.9	273.6	- 1.7	0.76	0.81	- 0.05	β , 1.
1883.78	269.7	280.2	- 10.5	1.07	0.79	+ 0.28	En., 5.
1885.77	284.9	286.6	1.7	...	0.76	...	H Σ , 1.
1886.03	270.1	287.6	- 17.5	1.03	0.75	+ 0.28	T., 4.
1888.70	298.1	297.4	+ 0.7	0.83	0.70	+ 0.13	β , 4.
1889.52	304.4	300.9	+ 3.5	0.76	0.67	+ 0.09	β , 3.
1890.62	308.6	305.8	+ 2.8	0.55	0.64	- 0.09	β , 4.
1891.61	313.5	310.6	+ 2.9	0.58	0.62	- 0.04	β , 4.
1892.99	317.2	316.4	+ 0.8	0.5 $\pm$	0.58	- 0.08	Sp., 1.
1893.47	322.2	321.2	+ 1.0	0.65	0.56	+ 0.09	Bax., 1.
1896.09	333.7	340.2	- 6.5	0.5 $\pm$	0.48	+ 0.02	Sp., 1.
1898.92	5.9	7.3	- 1.4	0.36	0.42	- 0.06	A., 3.
1899.72	11.6	16.3	- 4.7	0.42	0.41	+ 0.01	A., 3-2.
1900.76	29.3	28.2	+ 1.1	0.38	0.40	- 0.02	A., 1.
1904.00	65.2	67.3	- 2.1	0.40	0.40	$\pm$ 0.00	A., 2.
1906.32	97.4	95.5	+ 1.9	0.37	0.39	- 0.02	A., 2.
1907.93	118.8	115.9	+ 2.9	< 0.4	0.39	+ 0.01	Doo., 1.



Ephemeris.

t	θ_c	ρ_c	t	θ_c	ρ_c
1908'60	124°3	0''38	1914'60	193°4	0''47
1909'60	137°7	0'38	1915'60	201°2	0'50
1910'60	150°5	0'39	1916'60	208°2	0'54
1911'60	163°0	0'40	1917'60	214°2	0'57
1912'60	174°4	0'42	1918'60	219°8	0'60
1913'60	184°3	0'44			

All the observations are given in Burnham's *General Catalogue of Double Stars* recently published by the Carnegie Institution. It is sufficient, therefore, to add a comparison of the computed with the observed places upon which the apparent orbit was based.

The elements of β 513 are as follows:—

P =	52.95 years	Ω =	81°·4
T =	1905'60	i =	35°·9
e =	0'347	λ =	6°·2
a =	0''·610		

Apparent orbit:—

Length of major axis	=	1''·214
Length of minor axis	=	0''·962
Angle of major axis	=	84°·7
Angle of periastron	=	86°·7
Distance of star from centre	=	0''·21.

For several years to come β 513 will deserve careful attention from observers with powerful telescopes. The motion will be rapid for some ten years, after which the system will widen out, so that observation will be much easier than at present.

*U.S. Naval Observatory,
Mare Island, California,
1907 December 10.*

On the Orbit of $\beta 552 = 11$ Orionis.

By T. J. J. See, A.M., Ph.D. (Berol.).

This interesting binary was discovered by Burnham with the 18½-inch refractor at Chicago in 1877, but it had been previously suspected with the 6-inch in 1874 as "possibly a close pair." The place for the epoch 1880 is $\alpha = 4^h 45^m 4^s$; $\delta = +13^\circ 27'$; and the magnitudes are 7.0 and 10.0. It is therefore an exceedingly difficult object, and can be well seen only in the most powerful telescopes, under good conditions. On two favourable nights in 1892 Burnham could not see it with the 36-inch refractor of the Lick Observatory. Professor Hermann Struve, with the 30-inch refractor at Pulkowa, had suspected the companion on the opposite side in 1886, when the distance was only $0''.21$; but as his angle corresponds to the true place at the time, I have reversed it, on the supposition that the point of the diffraction fringe noted was produced by the companion, which was then too close for separation, but might easily produce an opposite image resembling a real companion.

This system is highly inclined upon our visual ray. The components being so unequal, the observations are somewhat discordant. After a careful study of the movement, I have deduced the following elements:—

P = 56.0 years	$\Omega = 4^\circ.1$
T = 1887.0	$i = 56^\circ.7$
$e = 0.345$	$\lambda = 90^\circ.2$
$a = 0''.528$	

Apparent orbit:—

Length of major axis	= $1''.00$
Length of minor axis	= $0''.59$
Angle of major axis	= $4^\circ.2$
Angle of periastron	= $94^\circ.4$
Distance of star from centre	= $0''.10$

Comparison of computed with observed places for
 $\beta 552 = 11$ Orionis.

t	θ_0	θ_c	$\theta_0 - \theta_c$	ρ_0	ρ_c	$\rho_0 - \rho_c$	Observer, Nights.
1874.95	$340^\circ \pm$	$348^\circ.6$	$- 8^\circ.6$	$0''.5 \pm$	$0''.51$	$- 0''.01 \pm$	β .
1877.97	$360^\circ \pm$	$359^\circ.5$	$+ 0^\circ.5$	$0''.8 \pm$	$0''.49$	$+ 0''.31 \pm$	β , 1.
1886.19	$85^\circ.0 ?$	$74^\circ.5$	$+ 10^\circ.5$	$0''.5 \pm$	$0''.21$	$+ 0''.29 \pm$	H 2, 1; angle is here reversed
1889.19	$144^\circ.7$	$141^\circ.2$	$+ 3^\circ.5$	$0''.35 \pm$	$0''.25$	$+ 0''.10 \pm$	Sp., 1.
1890.96	$156^\circ.7$	$161^\circ.4$	$- 4^\circ.7$	$0''.33$	$0''.34$	$- 0''.01$	β , 3.
1891.13	$153^\circ.8$	$162^\circ.8$	$- 9^\circ.0$	$0''.33$	$0''.34$	$- 0''.01$	Sp., 8.
1892.06	...	$169^\circ.3$	...	...	$0''.38$	...	β , 2; apparently single with 36-inch.

t	θ_0	θ_c	$\theta_0 - \theta_c$	ρ_0	ρ_c	$\rho_0 - \rho_c$	Observer, Nights.
1893'07	165°3	175°5	-10°2	0"35±	0"42	-0"07	Sp., 1.
1894'15	178°2	181°1	-2°9	0'38	0'45	-0'07	Sp., 2.
1894'88	177°0	184°3	-7°3	0'40	0'47	-0'07	Bax., 2.
1896'11	179°9	188°8	-8°9	0'35	0'49	-0'12	Sp., 7.
1896'86	187°0	192°1	-5°2	0'42	0'50	-0'08	Hu., 1 ; A., 3.
1897'63	193°5	194°9	-1°4	0'37	0'50	-0'13	Sp., A., 5 ; Hu., 3.
1898'01	189°9	197°7	-7°8	0'45	0'50	-0'05	Lewis, 2.
1898'43	198°1	197°8	+0°4	0'44	0'51	-0'07	A., 6.
1899'76	202°5	202°0	+0°5	0'45	0'51	-0'06	See, 1 ; A., 4.
1899'85	207°3	202°8	+4°5	0'42	0'51	-0'09	Lewis, 3.
1900'71	204°3	205°9	-1°6	0'45	0'51	-0'06	A., 3 ; Doo., 3.
1901'85	204°6	210°1	-5°5	0'49	0'50	-0'01	A., 2 ; Doo., 2.
1902'74	229°3	212°6	+16°7	0'32	0'50	-0'18	β , 1.
1903'66	218°9	216°8	+2°1	0'46	0'49	-0'03	A., 4 ; Biesbroeck, 2.
1905'90	227°9	225°9	+2°0	0'46	0'46	±0'00	A, 2.
1907'93	238°7	235°1	+3°6	0'55	0'44	+0'11	Doo., 3.

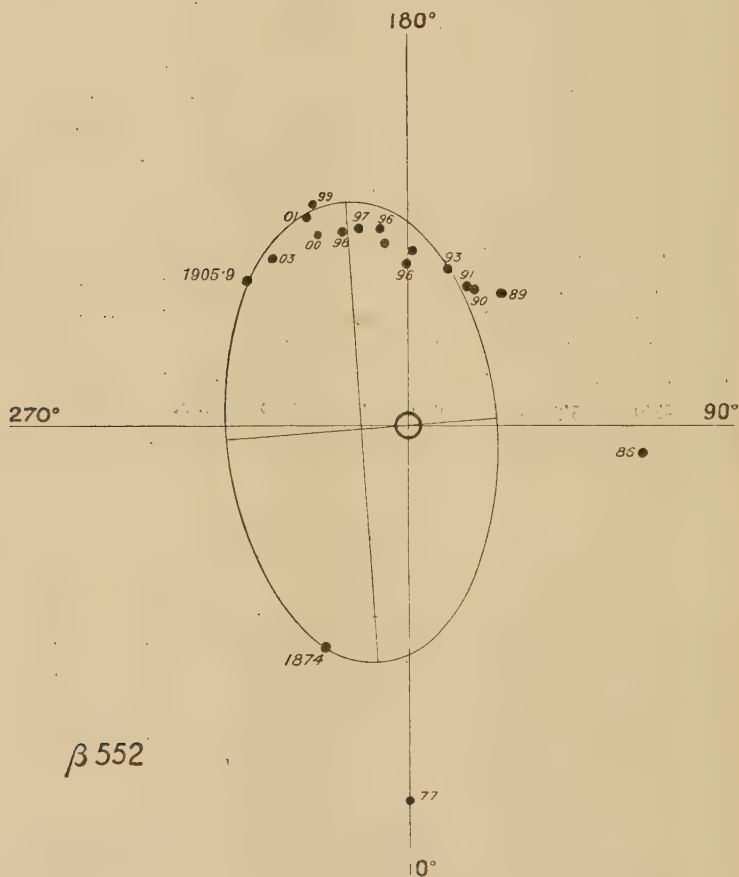
Ephemeris.

t	θ_c	ρ_c	t	θ_c	ρ_c
1907°0	230°9	0"45	1913°0	262°3	0"39
1908°0	235°5	0'44	1914°0	268°4	0'39
1909°0	240°5	0'43	1915°0	274°5	0'39
1910°0	245°6	0'42	1916°0	280°6	0'39
1911°0	250°6	0'41	1917°0	286°5	0'39
1912°0	256°3	0'40	1918°0	292°4	0'40

The accompanying table shows the comparison of the computed with the observed places. Considering the discordance of the measures, the representation in angle is good. The distances could be slightly improved by reducing the major axis; but as the scale of distance is still somewhat uncertain, I have not thought it worth while to do this. It seems certain that these elements give the essential character of the motion, which is rendered very peculiar by the high inclination, giving a nearly stationary phase at each end of the apparent orbit. The orbit of β 552 has some resemblance to that of γ Coronæ Borealis, the components of which are correspondingly unequal and difficult to measure.

It is clear that the periastron was swept over about 1887°0, and that the period is about fifty-six years. The eccentricity may be higher than that given here, but an increase in this element

would hardly be warranted by the existing observations. The ephemeris will be useful to observers, and it is to be hoped that



they will continue to follow this interesting object for a number of years, till the orbit can be adequately confirmed.

*U.S. Naval Observatory,
Mare Island, California,
1907 December 12.*

Note on the Comparative Eccentricities of Visual and Spectroscopic Binary Stars. By T. J. J. See, A.M., Ph.D. (Berol.).

For some time the writer has been occupied with a critical examination of the comparative eccentricities of visual and spectroscopic binary stars, in the hope of obtaining an observational criterion that will throw light on the evolution of these systems. It is now clear that the investigation will afford the criterion sought; and as he recently had occasion to mention the general result in letters to some friends abroad, it seems likely that a preliminary notice of this work may be of interest to the Society.

If we exclude from consideration all binaries telescopically resolvable in modern instruments, there remain about 30 orbits of purely spectroscopic binaries, which have been worked out from spectrograms taken chiefly at the Lick, Yerkes, and Potsdam Observatories. The eccentricity is well determined, and the average value for these 30 systems is about 0.22. In the case of very rapid binaries, with periods often much less than a day, the orbits are practically circular. Dr. A. W. Roberts and Professor G. W. Myers have investigated the light changes of certain variables, and shown that they are composed of two stars in the process of separation, and therefore revolving in orbits which are without sensible eccentricity.

The average eccentricity of visual binary stars remains about 0.50, as indicated in the work *Researches on the Evolution of the Stellar Systems*, vol. i., published in 1896. The subject has, however, been so much extended and improved during the past eleven years that it has seemed advisable to re-examine each orbit anew. There are about 80 orbits in which the eccentricity may be fairly well determined. I hope to have the final result of this examination ready for presentation to the Society in a short time. The work is already so far advanced that we may confidently state that *the average eccentricity among visual binaries is considerably more than twice that among the spectroscopic binaries.* The latter are stellar systems, with components so close together, owing to the small size of their orbits, or so far removed from us in space, that we cannot resolve them even in the greatest of modern telescopes. In some cases they are closer sub-systems of known visual binary stars. Among such systems we may mention ξ Ursae Majoris, κ Pegasi, α Geminorum, etc., which are spectroscopically subdivided in exact analogy to the many double stars which have been resolved into triple stars by increase of modern telescopic power; but where the spectroscopic binaries stand alone, we can infer the average size of their orbits only from the generalised form of Kepler's law.

For a large number of systems we are justified in assuming that the average mass will be the same among the spectroscopic as among the visual binary stars. And as the periods of the spectroscopic binaries are much shorter than those of the visual binaries, it

follows also that their mean distances are correspondingly smaller. This conclusion might indeed fail here and there in particular cases, but, as applied generally to these two classes of bodies, is entirely valid.

Accordingly, we see that in the universe generally small eccentricity is connected with the small mean distance, and *vice versa*. This confirms, from an observational standpoint, the theory of the development of these systems long ago inferred from other considerations (*cf.* the writer's *Inaugural Dissertation*, Berlin, 1892). Heretofore scarcely anyone has dared to hope that a direct confirmation of this theory would ever be possible; but, unless this reasoning is entirely at fault, such a positive confirmation is near at hand, and is due to the notable advance in our knowledge of the stars arising from the discovery and investigation of spectroscopic binaries, chiefly during the past ten years.

*U.S. Naval Observatory,
Mare Island, California:
1907 November 30.*

Micrometrical Measures of Double Stars (Fifth Series). By the
Rev. T. E. Espin, M.A.

In the early part of 1907 very few known double stars were measured, pending the arrival of Burnham's General Catalogue. The latter part of the year has been devoted to the completion of measures of new pairs, and to the measurement of such stars of Sir John Herschel's as have not been remeasured since his observation of them. The places are those of the General Catalogue (1880); but in several cases, as explained in the notes, corrections to Herschel's places have been made.

<i>h.</i>	<i>R.A. 1880. Decl.</i>		<i>P.</i>	<i>D.</i>	<i>Mags.</i>	<i>Nights.</i>	<i>Date.</i>
	^{<i>h</i>}	^{<i>m</i>}					
1009	0	9 ⁶	+47 56	26 ⁷	16 ⁷⁷	9 ⁰ , 9 ¹	2 1907-760 AB
				140 ⁹	36 ¹⁵	11 ⁷	2 760 AC
1029	25 ⁹	44	16	279 ⁹	11 ⁸⁵	8 ⁸ , 10 ⁴	3 905
1985	28 ⁴	48	11	143 ³	17 ⁹⁷	10 ⁰ , 10 ²	2 851
1049	38 ⁰	50	6	303 ¹	15 ³²	9 ² , 11 ⁰	3 784
1050	38 ¹	44	23	185 ¹	13 ⁹⁷	9 ⁷ , 10 ⁷	2 896
1058	51 ⁰	49	34	283 ⁷	11 ⁷⁰	10 ² , 10 ³	3 862
1060	52 ⁰	44	16	293 ¹	9 ⁴⁸	9 ⁵ , 10 ⁷	4 924
2013	59 ²	44	8	247 ⁰	23 ²²	8 ⁵ , 11 ⁵	2 891
2031	1	9 ⁶	43 49	260 ⁶	20 ¹⁵	8 ⁵ , 12 ⁷	2 903 BC
				342 2	45 ⁵²	A = 8 ⁰	2 903 AB

h.	R.A. 1880.		Decl.		P.	D.	Mags.	Nights.	Date.
	h	m	°	'					
1077	1	10.7	+44	0	295.2	38.05	8.0, 9.6	2	1907.903
2144	2	25.0	48	20	259.4	26.22	8.6, 9.5	2	.880
2232	4	22.4	47	2	325.3	13.42	8.8, 10.5	2	.924
2237		36.1	47	26	123.0	15.90	9.1, 11.6	2	.942
2239		39.4	45	58	162.2	13.52	9.5, 10.5	2	.942
2248		58.2	47	11	333.1	14.92	8.8, 11.0	2	.994
2249	5	1.1	47	21	97.2	14.20	8.8, 9.4	2	.942
2264		21.7	47	49	125.8	5.70	8.4, 9.5	2	.942
378	6	0.2	28	58	257.2	12.27	9.0, 9.0	2	.115
379		1.3	31	17	117.3	9.85	7.0, 11.5	1	.041
460	8	45.3	28	43	333.7	43.72	7.5, 11.0	2	.183
803		56.7	28	4	9.4	11.62	9.2, 11.5	2	.232
495	9	12.9	35	46	135.2	15.81	11.0, 11.0	2	.216
2857	19	7.0	41	35	177.5	14.40	8.0, 13.5	1	.756 AB
					210.9	36.60	12.0	1	.756 AC
1460		53.6	46	28	87.2	6.87	9.9, 11.0	2	.835
1463		55.9	45	29	313.1	10.20	9.8, 11.5	3	.897
1481	20	1.8	49	3	7.2	17.40	10.0, 10.1	2	.842 BC
					88.0	27.62	A = 9.7	2	.842 AB
1584		45.6	47	38	224.0	3.75	9.0, 10.0	2	.770
1594		50.4	47	6	48.6	8.78	9.0, 10.1	4	.878
1604		56.1	48	43	135.5	25.67	10.2, 11.0	2	.825
1651	21	24.5	47	38	230.0	10.95	9.4, 10.1	2	.828
1657		28	47	54	See Note				
1676		35.1	46	39	135.0	29.22	8.4, 9.7	2	.926
1681		35.6	47	52	109.9	7.35	9.0, 9.1	2	.770
1692		42.0	46	39	249.6	11.75	10.5, 11.5	3	.930
1698		44.9	46	43	325.6	7.62	9.1, 12.0	3	.930
1701		46.9	46	32	195.3	11.42	9.7, 10.7	2	.951
1705		49.8	46	29	37.6	3.07	11.9, 13.5	2	.904
1712		55.3	48	8	176.7	9.30	9.4, 9.5	3	.786
1738	22	4.5	45	53	178.9	9.57	10.6, 11.7	2	.932
1762		18.5	47	48	88.4	14.70	9.7, 13.7	2	.770 BC
					354.9	18.35	A = 9.1	2	.770 AB
1766		21.8	49	41	265.2	13.32	9.0, 10.1	2	.806

<i>h.</i>	R.A. 1880.	Decl.	P.	D.	Mags.	Nights.	Date.
1772	^h 22 ^m 23 ^s ·5	+45 32	103 ^s ·5	6 ^s ·05	9 ^s ·6, 10 ^s ·2	2	1907·910
1793	33 ^s ·1	46 25	291 ^s ·5	13 ^s ·95	10 ^s ·0, 11 ^s ·7	2	·910
1794	33 ^s ·2	46 22	315 ^s ·0	21 ^s ·80	9 ^s ·4, 11 ^s ·0	2	·910
1795	33 ^s ·3	46 43	213 ^s ·8	12 ^s ·07	10 ^s ·7, 11 ^s ·5	2	·910 BC
			209 ^s ·8	16 ^s ·10	A = 10 ^s ·2	2	·910 AB
1797	34 ^s ·6	49 30	125 ^s ·4	14 ^s ·10	9 ^s ·7, 11 ^s ·0	2	·951
1814	44 ^s ·1	47 57	76 ^s ·1	14 ^s ·10	9 ^s ·0, 10 ^s ·2	2	·782
1863	23 10 ^s ·4	48 21	51 ^s ·1	8 ^s ·57	11 ^s ·5, 12 ^s ·5	3	·813
1878	21 ^s ·3	49 46	268 ^s ·2	8 ^s ·87	10 ^s ·5, 10 ^s ·7	2	·858
1883	21 ^s ·9	45 44	145 ^s ·4	18 ^s ·55	9 ^s ·3, 9 ^s ·6	2	·903
1893	29 ^s ·2	46 50	247 ^s ·0	4 ^s ·47	8 ^s ·6, 8 ^s ·7	3	·795
1917	48 ^s ·7	45 6	76 ^s ·2	12 ^s ·92	8 ^s ·9, 12 ^s ·2	2	·935
1923	54 ^s ·3	50 3	277 ^s ·6	10 ^s ·77	11 ^s ·5, 11 ^s ·8	2	·918
1927	57 ^s ·0	44 28	76 ^s ·6	14 ^s ·75	8 ^s ·6, 8 ^s ·9	2	·951
1931	58 ^s ·0	49 18	113 ^s ·1	21 ^s ·90	8 ^s ·0, 10 ^s ·7	2	·759

Notes.

- h* 1009 is BD + 47°, 40, C not given by *h*.
h 1029 is BD + 44°, 110, *h*'s place one min. too small in R.A.
h 1985 is BD + 48°, 172.
h 2013 is BD + 44°, 232, *h*'s angle 256°·0.
h 2249 a small star at P 306°, D 119" is double and is No. 467 of the list of new pairs.
h 2857 the nearer *comes* is not in *h*.
h 1460 22^s·2 *p* 43^s·7 N of BD + 46°, 2817.
h 1657 *h* says "10° ± 10" ± in a cluster." I have looked for this on several nights, but have failed to find any such pair. The only pair that bears any resemblance is one at P 16°·3 D 101" from BD + 47°, 3439 (Aitken 770) which I have measured as follows:—
P 39°·7, D 7"·00, mags. 9·5, 10·7, 2 nts. 1907·884.
h 1705 This is 13^s·6 *f* 2^s·5 N of BD + 46°, 3480, and if identical with *h* 1705 *h*'s place is 1 min. too great. His observation is "P 80°·4, D 3½", mags. 11–12, 14, difficult." I could not find any other pair here.
h 1762 Star C not given by *h*.
h 1795 *h* says, "if a 4th star." No such star seen on either night, save a distant brighter one n.p.
h 1863 *h* reversed angle and says "difficult to measure."
h 1893 This is BD + 46°, 4082, and *h*'s place requires a correction in Decl. of + 30'. His angle is the mean of 250°·6 and 252°·6. The star BD + 46°, 4083, is in the field 8^s·8 *f* 73"·45 N at an angle of 51°·6.

Various Stars.

Name.	R.A. h m	1880. Decl.	P.	D.	Mags.	Nights.	Date.
η Cassiopeiæ ...	0 41'7	+57 11	236'4	5'67		2	1907'043
			236'1	5'72		2	'990
Espin 44 ...	49'8	56 51	250'1	7'40	8'3, 12'0	3	'341
ϕ Cassiopeiæ...	1 12'5	57 36	264'8	41'54	7'0, 11'0	2	'107 CD
			231'1	132'93	A= 5'0	2	'107 AC
A.G. (Bonn) ...	51'7	50 1	355'2	13'62	9'0, 9'1	2	'774
A.G. (Bonn) ...	3 17'3	46 36	315'0	10'67	8'5, 9'2	2	'957
Espin 57 ...	4 47'4	47 27	175'9	2'27	9'5, 9'9	3	'942 AB
			61'3	16'52	C= 14'0	2	'940 AC
A 1316 ...	5 54'5	30 0	181'0	4'68	8'5, 11'7	3	'121
Rümker 4 ...	6 12'8	29 43	273'2	5'50	9'0, 9'2	2	'115
6 Cancri ...	7 56'8	28 8	187'5	60'06	5'0, 12'0	2	'232 AB
			81'0	78'63	C= 11'0	2	'232 AC
Σ 1321 ...	9 6'4	53 13	66'5	18'82		2	'202 AB
			282'7	28'44	C= 14'0	2	'202 AC
Küstner 40 ...	11 42'6	34 22	182'6	3'12	8'9, 9'2	2	'216
12 Comæ ...	12 16'5	26 31	54'0	34'91	4'5, 11'5	3	'240 AB
			167'5	64'66	C= 8'0	2	'235 AC
A.G. (Bonn) ...	18 54'7	47 2	229'0	2'37	8'3, 8'4	3	'846
Lewis 32 ...	19 56'1	29 36	128'0	3'17	9'8, 10'0	3	'724
Arg. 39 (Espin 20 92)	38'7	48 50	113'2	9'67	8'2, 8'6	3	'778 AB
			125'1	30'98	C= 12'0	4	'714 BC
			232'0	33'47	D= 14'0	2	'893 BD
+ 48°3995 ...	23 13'5	48 16	231'6	49'10	7'3, 8'8	2	'799 -

Notes.

ϕ Cassiopeiæ CD may be the pair measured by H as $271^{\circ}8$.

A. G. III. 17'3. The note in Bonn Observations "Dupl. 3" med." may refer to Σ 371, 7 min. p.

A 1316. Detected and measured before Professor Aitken's results reached me. He calls the *comes* 14'2 mag.

Rümker 4. This is BD + 29°, 1181, 6' N of Rümker's place.

6 Cancri. A star mentioned in "Celestial Objects" as having two *comites* detected by Birmingham.

Σ 1321. The minute *comes* was first seen 1901 Jan. 22.

12 Comæ. The nearer *comes* previously seen by Burnham.

Lewis 32. This is BD + 29°, 3849.

Arg. 39. After charting all the available measures, I find a motion for B of $0''.216$ towards $221^{\circ}5$. C, measured by Burnham, is almost at right angles to the line of motion. I have added the still fainter star D, as being nearly in the direction of motion.

New Double Stars. By the Rev. T. E. Espin, M.A.

No.	B.D.	R.A. 1900. h m	Decl. ° ' "	P.	D.	Mags.	Nights.	Date.
								1907.
443	+48°5	0 0'8	+49°10'	32°9	4"27	8'7 9'9	2	'740
444	+44,126	31'4	44 57	192°5	2'93	9'1 9'6	3	'968
445	+44,130	32'6	44 54	349°7	3'32	9'0 11'0	3	'968
446	+49,182	40'2	49 18	256°1	10'12	8'4 9'0	2	'867
447	+49,186	41'1	49 43	277°9	5'60	8'5 11'7	2	'836
448	...	59'1	50 2	81°3	2'50	9'5 9'7	2	'774
449	+49,352	1 13'8	49 58	212°8	11'20	8'5 9'1	3	'876
450	+47,414	21'7	47 37	147°4	9'52	8'0 13'5	2	'759 AB
				268°9	20'10	C = 13'0	2	'759 AC
451	+49,386	21'8	49 57	65°1	1'97	9'1 9'2	3	'886
452	+49,420	31'7	50 8	176°2	2'62	9'4 11'3	2	'881
453	+44,387	48'9	44 50	259°0	4'45	8'5 11'2	2	'994
454	+49,514	53'2	50 7	132°2	8'65	8'5 10'0	3	'786
455	+49,637	2 14'0	49 41	135°6	4'54	9'2 13'0	2	'850
456	+49,665	21'0	50 2	269°8	3'28	9'1 11'4	3	'821
457	+49,671	22'2	49 14	6°7	6'80	7'7 11'5	2	'759
458	+48,701	28'1	48 58	316°8	5'30	9'1 9'9	3	'886 AB
				243°9	24'58	C = 13'5	3	'886 AC
459	+48,708	29'9	48 56	142°7	3'35	8'9 10'2	4	'888
460	+48,711	30'9	48 13	316°8	2'68	9'1 9'3	3	'894 AB
				336°8	18'85	C = 13'5	1	'909 AC
461	+48,756	40'2	48 40	342°7	8'22	8'5 9'6	3	'900
462	...	3 7'9	49 23	172°9	2'55	9'7 10'4	2	'896
463	+49,891	9'5	49 21	257°3	4'95	9'3 11'7	2	'896
464	+47,806	14'2	47 21	65°8	7'15	9'1 9'2	2	'817
465	+49,1015	38'6	50 7	251°1	7'37	8'7 9'2	2	'896
466	+49,1092	56'7	49 32	57°1	3'95	8'5 11'0	2	'896 AB
				123°4	14'22	C = 12'0	2	'896 AC
467	...	5 2'4	47 25	98°2	1'52	11'0 11'3	2	'940
468	+30,3021	17 30'3	30 3	11°5	2'28	9'2 10'0	4	'613
469	+28,2829	42'8	28 1	140°9	4'41	10'0 13'0	3	'650 BC
				267°5	48'12	A = 8'5	2	'658 AB
470	+27,2943	59'3	27 51	206°5	7'17	8'6 9'8	3	'658
471	+27,2948	18 0'7	27 6	271°4	18'23	7'0 14'0	3	'676 AB
				44°1	30'35	C = 10'0	2	'671 AC
472	+27,2980	9'9	27 57	8°5	3'08	9'1 9'6	2	'655
473	+42,3026	10'0	42 49	158°0	9'63	8'5 10'0	3	'703 BC
				100°3	30'78	A = 8'5	3	'703 AB

No.	B.D.	R.A. 1900.	Decl.	P.	D.	Mags.	Nights.	Date.
		h m						
474	+42,3054	18 16.8	+42 55	34.5	3.68	9.2 12.3	3	1907. '725
475	+27,3032	24.0	27 2	222.9	9.72	8.8 9.8	2	'671
476	+27,3041	26.7	28 0	347.6	2.52	9.5 10.1	3	'666
477	+26,3298	30.9	26 56	233.1	7.76	8.9 11.5	3	'677
478	+42,3144	40.5	42 30	182.3	8.22	8.7 9.2	2	'699
479	+27,3208	57.9	27 9	139.3	8.05	8.6 9.8	2	'603
480	+27,3218	58.9	27 33	304.2	2.45	9.1 10.1	2	'637
481	+27,3261	19 6.2	27 41	136.0	4.32	9.0 12.7	3	'650
482	+25,3756	9.7	25 36	167.4	4.27	8.9 11.2	2	'609
483	+25,3803	17.5	25 23	358.2	12.62	7.2 12.0	4	'634
484	+28,3322	20.8	28 44	332.6	7.30	8.5 14.0	3	'692 AB
				335.6	22.60	C = 9.0	3	'692 AC
485	+27,3419	27.5	27 55	342.4	2.37	9.2 10.5	2	'655
486	+25,3870	28.6	25 13	287.9	8.35	9.0 14.0	3	'656 BC
				18.2	45.24	A = 8.8	2	'633 AB
487	+27,3431	29.8	27 10	336.4	3.87	9.1 11.5	2	'708
488	+27,3434	29.9	27 16	50.5	2.12	9.1 9.2	2	'708
489	+27,3438	31.4	27 41	243.2	15.40	8.2 10.8	3	'714 AB
				205.5	20.32	C = 11.2	3	'714 AC
490	+43,3305	33.4	43 14	166.8	3.50	9.0 10.2	2	'740 BC
				229.7	16.38	a = 12.0	3	'758 Aa
				223.8	61.70	A = 8.9	2	'740 AB
491	+47,2874	33.5	47 54	57.7	9.67	9.1 9.6	2	'720 AB
				251.0	18.20	C = 14.0	1	'725 AC
492	+25,3902	34.2	25 28	158.1	5.42	9.2 9.3	2	'632
493	+43,3311	34.5	43 14	316.0	4.20	9.3 9.5	3	'758
494	+27,3549	49.6	27 49	190.1	3.65	8.9 10.2	2	'725 AB
				309.0	16.35	C = 12.0	1	'739 AC
495	+28,3553	54.3	28 43	310.5	4.11	11.0 11.8	3	'657 BC
				245.5	39.87	A = 8.0	2	'688 AB
496	+28,3594	58.5	28 42	279.2	2.22	8.9 10.0	2	'688
497	+29,3886	20 1.1	30 4	200.3	5.28	9.1 10.8	3	'658
498	+29,3889	1.7	30 6	241.3	9.38	8.3 11.2	3	'658
499	+45,3080	7.8	45 18	339.8	14.35	8.0 12.0	3	'784 AB
				77.3	26.20	C = 13.0	1	'756 AC
500	+49,3218	8.8	49 25	351.6	6.45	8.8 9.2	3	'891
501	+45,3093	9.0	45 17	125.3	1.32	9.1 10.2	2	'715
502	+48,3059	10.9	48 53	223.2	11.25	7.7 10.0	4	'897
503	...	16.5	47 48	353.1	4.57	9.2 9.5	3	'734
504	+28,3760	24.1	28 40	87.2	6.02	9.2 10.2	2	'783

No.	B.D.	R.A. 1900.	Decl.	P.	D.	Mags.	Nights.	Date.
		h m						1907.
505	...	20 26.5	+30 9	13.0	1.92	9.8	10.0	2 783
506	+28,3779	27.1	28 35	303.1	6.57	8.7	11.2	2 654
507	+28,3790	28.9	28 31	295.5	7.30	8.5	11.0	2 603 AB
				340.2	32.73	C =	8.6	2 603 AC
				224.3	36.52	D =	9.1	2 603 AD
508	+28,3847	36.9	29 4	230.1	2.25	8.6	10.0	2 783 AB
				221.9	18.07	C =	10.0	2 783 AC
509	+48,3187	37.5	48 39	283.6	7.22	8.8	12.0	3 723
510	+27,3871	43.9	27 31	235.1	6.11	10.5	10.7	3 704 BC
				343.3	43.45	A =	8.0	3 704 AB
511	+26,4026	50.9	26 42	51.5	7.42	9.0	9.7	2 640
512	+46,3201	21 5.5	46 52	18.3	13.27	7.0	12.3	3 710
513	+45,3474	12.3	45 59	52.2	7.05	9.0	13.5	2 847
514	+46,3246	12.3	46 54	195.9	3.85	8.9	9.3	3 695
515	+25,4504	14.0	25 49	302.5	3.50	9.2	12.7	2 721
	+48,3376	21.4	48 53	161.5	19.16	6.5	12.0	3 709
516	...	26.7	46 51	140.5	3.31	9.4	11.0	3 844
517	+28,4122	28.4	29 4	239.4	2.05	9.2	9.9	4 781
518	+45,3589	30.6	46 3	11.9	4.55	9.2	12.3	3 666
519	+47,3476	30.6	47 42	304.3	7.62	9.5	9.7	2 801
520	+27,4125	34.6	27 59	38.3	7.19	9.4	9.9	4 714
521	+28,4171	39.9	28 19	269.7	14.25	7.6	14.0	2 628
522	+44,3972	48.7	44 46	174.7	4.35	9.0	11.0	2 888
523	+26,4294	49.1	26 33	94.9	6.19	9.1	9.2	3 764
524	+48,3536	49.6	48 52	5.5	5.87	11.5	13.3	3 776 BC
				188.7	20.37	A =	9.0	2 711 AB
525	+48,3544	50.9	48 48	149.1	3.82	8.6	10.5	2 711
526	+25,4650	52.0	25 44	108.4	7.34	8.4	12.5	3 771
527	+27,4230	56.8	27 21	207.4	3.05	9.7	9.9	2 674
528	+46,3546	58.6	46 44	74.5	2.47	9.1	11.7	2 720
529	+46,3555	59.4	46 17	71.6	5.75	8.5	11.0	2 705
530	+47,3677	59.5	48 11	190.9	8.17	8.0	9.5	2 703
531	+47,3680	22 0.2	48 10	225.1	2.55	9.2	11.5	3 706
532	+46,3580	2.4	46 47	245.6	9.82	9.0	9.1	2 720
533	+45,3848	10.8	45 23	321.5	4.25	9.0	12.0	2 730
	+48,3665	13.5	48 39	292.4	21.22	7.0	12.0	2 755
534	+48,3673	14.4	49 10	104.5	6.02	9.0	9.3	2 789 BC
				248.7	9.37	a =	12.0	2 789 Aa
				68.0	48.97	A =	8.5	2 789 AB
535	+25,4724	20.0	25 39	301.9	8.33	9.3	10.8	3 828

No.	B.D.	R.A. 1900.		Decl.	P.	D.	Mags.		Nights.	Date.
		h	m							
536	...	22	20.4	+26 54	270.1	2.97	10.2	10.2	2	1907. '673
537	+49,3855		23.2	50 9	19.7	2.27	9.2	9.4	2	'806
538	+46,3716		24.7	46 24	66.8	3.40	9.1	10.5	2	'889
539	+42,4437		28.3	42 36	287.7	6.10	9.1	11.0	3	'836
540	+48,3762		28.7	48 26	281.5	1.30	9.1	9.9	3	'709
541	+25,4787		35.9	26 13	220.3	6.61	8.0	12.4	4	'751
542	+49,3968		48.9	49 29	282.6	5.05	8.7	10.5	3	'710 AB
					83.9	57.88	C =	8.8	2	'708 AC
543	+46,3945	23	3.8	46 58	22.0	3.87	9.2	10.5	2	'783
544	+49,4054		4.2	49 22	245.6	2.30	9.5	9.7	2	'951
	+25,4927		17.5	25 22	264.4	19.95	6.7	13.0	2	'713
545	+48,4024		18.9	48 44	319.7	9.70	8.1	12.0	3	'755
546	+26,4623		20.1	26 24	162.2	2.37	9.1	10.8	3	'683
547	+46,4096		31.9	46 46	237.1	6.95	8.5	12.0	2	'795
548	+46,4139		39.8	46 54	315.9	14.37	8.5	8.8	2	'835
549	+47,4264		40.0	47 57	51.7	8.10	8.5	13.7	2	'704 AB
					238.7	16.07	C =	9.0	2	'704 AC
550	+45,4323		41.8	46 7	167.2	17.35	8.6	8.9	2	'814
551	+47,4313	23	49.5	47 41	311.6	4.72	11.7	12.7	2	'834 BC
					11.4	24.60	a =	10.5	1	'865 Aa
					88.1	30.90	A =	8.6	2	'834 AB

*Appearance of Ring-extension during the disappearance of
Saturn's Ring, Oct.-Nov. 1907. By R. T. A. Innes.*

As Professor Campbell's observation of the appearance of Saturn's ring will give rise to some discussion, I do not delay sending the notes I have made so far. The telescope used is the 9-inch Grubb refractor mentioned in an earlier note. Sir Howard Grubb has supplied it with a double concave eyepiece of high magnifying power, which transmits at least twice as much light as an ordinary eyepiece. This negative eyepiece gives an erect image; its only drawback is an excessively small field.

On several occasions Mr. R. N. Kotze, my son and myself noticed the beaded appearances referred to by Professor Campbell, but we assumed that they were caused by either Mimas or Enceladus, and our curiosity did not lead us further.

I now give, without further comment, the exact notes made by myself at the telescope:—

1907 Oct. 16. Ring seen at moments.

17. Ring all but held steadily. It is very faint and not full length.

- 1907 Oct. 21. Extension of ring clearly seen on f. side; not so distinct on p. side, but Titan and other satellites near.

Although I refer to the appearance as the "Ring" on the 16th and 17th, this word might convey a wrong impression. What is seen is a ghost-like extension where the ring was. It tapers off sharply, and is perhaps shorter than the ring.

22. to Nov. 20. Between these dates this ghost-like ring was seen on all occasions that the planet was looked at.
- Nov. 21. Planet just past meridian. Ring-extension seen to full length on each side. It is brightest close to the planet, and then it is discontinuous. Two cloud-bands on each side of equator. Shadow of ring on planet is a deep brown, and there is a suspicion, but nothing more than a suspicion, of its being divided into two lines. (On Nov. 6 it was noticed that the shadow of Titan was much darker than the ring-shadow.)
25. Ring-extension decidedly discontinuous, and doubtful if full length. The satellites no longer bead the ring-extension, but pass N or S of it.

A rough sketch will give an idea of the relative darkness of the sky, the ring-shadow, cloud-bands, and of the discontinuity of the ring-extension about the dates Nov. 22-25.

Professor Campbell's message reached here on the 28th inst.

- Dec. 1. Ghost-like extension, including discontinuities, is about half the diameter of Saturn on each side, and is discontinuous. It leaves the disc on the south side of the ring-shadow.

Johannesburg:
1907 December 2.

Observations of Saturn's Ninth Satellite, Phœbe, from Photographs taken with the 30-inch Reflector at the Royal Observatory, Greenwich, in 1907.

(Communicated by the Astronomer Royal.)

Phœbe has been under observation during the recent opposition of Saturn with the 30-inch reflector, photographs having been obtained whenever possible. In all 16 photographs have been secured on 16 nights, between August 10 and December 6.

From one to two hours' exposure was necessary on account of the faintness of the satellite and its low altitude (5° south of the equator).

The positions of the satellite have been measured on the photographs taken with the reflector with reference to three or four faint comparison stars (of eleventh or twelfth magnitude), symmetrically distributed about the satellite. The positions of these faint comparison stars were then measured relatively to the reference stars (of eighth to ninth magnitude) in the Astronomische Gesellschaft Catalogue on photographs (with 30^m and 40^m exposure) taken with the Astrographic 13-inch refractor. The field, sensibly free from distortion, being much larger with this telescope than with the reflector, from 16 to 20 reference stars were available on each plate.

As Saturn moved slowly, it was possible to make one reference plate serve for several photographs which were each referred to it. The constants were determined in the usual manner, all the stars on the plate given in the A.G. Catalogues being used for this purpose. Right Ascensions and Declinations of Phœbe were then determined and compared with the tabular positions of Saturn after applying to the tabular places the corrections

$$\text{R.A.} + 8.051 \quad \text{Dec.} + 0''.48$$

as determined below.

Observations of Phœbe.

	Date and G.M.T.				Apparent R.A.			Apparent Dec.			Phoebe - Saturn		Exp.			
	d	h	m	s	h	m	s	°	'	"	R.A. m	s		Dec. m		
Aug.	10	12	55	35	23	50	53.23	-3	34	29.7	-0	27.31	-4	22.0	105	
	11	12	39	47		50	41.08		36	0.6	0	28.71	4	29.5	133	
	18	12	39	10		49	7.84		47	44.6	0	38.25	5	39.8	143	
	19	12	53	50		48	53.36		3	49	32.2	0	39.62	5	50.1	120
	29	11	4	59		46	19.31		4	8	4.7	0	52.53	7	22.6	93
Sept.	2	10	56	11		45	12.25		15	54.4	0	57.50	7	55.9	54	
	9	11	45	18		43	9.46		30	1.9	1	5.68	8	53.6	107	
	10	11	47	20		42	51.68		4	32	2.2	1	6.74	9	0.2	70
Oct.	2	10	36	20		36	21.70		5	14	36.1	1	27.68	11	16.1	90
	4	10	52	26		35	47.97		18	7.7	1	29.11	11	24.7	120	
	8	10	44	28		34	42.92		24	51.1	1	31.83	11	40.2	53	
	12	9	1	36		33	41.99		31	1.8	1	34.22	11	52.4	90	
	30	8	33	44		29	55.18		52	45.3	1	40.62	12	18.1	100	
Dec.	3	6	34	27		28	4.66		56	56.9	1	33.00	10	45.9	121	
	5	6	35	9		28	12.86		55	35.8	1	31.77	10	36.3	120	
	6	6	45	38	23	28	17.69		5	54	49.7	-1	31.04	-10	30.0	90

Errors of Tabular Place of Saturn.

The positions of Phœbe given above depend on the positions of certain stars taken from two catalogues of the *Astronomische Gesellschaft*, and will be affected by the mean error in the places of these stars. To eliminate this it is necessary to determine the position of Saturn referred to the same stars.

For this purpose a series of photographs of Saturn was taken with the Thompson 26-inch refractor, using the occulting shutter. With a rather slow plate an exposure of 2 minutes was given to the stars in the field, Saturn being exposed for about $\frac{1}{100}$ th second every alternate second. In this way a good measurable disc of Saturn was obtained, whilst giving sufficient exposure for the stars.

The field of the 26-inch refractor on a 16 c.m. plate being only 1° square, or a quarter the area of the Astrographic plate, only three or four stars were usually available on each plate, but the positions of these were deduced from measures made on the Astrographic reference plates for Phœbe, using all the stars. The mean systematic error of these three or four stars will therefore be the same as if all the stars had been used, and the position of Saturn deduced will be affected by the same error, and will then be comparable with the deduced places of Phœbe.

Eleven photographs were selected for measurement. From two to four images of Saturn, and of each of the stars, were measured on each plate in the Astrographic micrometer, and the following are the results obtained. The observed R.A. and Dec. of Saturn were computed to $0^h 00^m 1$ and $0'' 01$ respectively; but as the tabular place in the N.A. is only given to $0^s 01$ and $0'' 1$ respectively, the errors are only given to that order of accuracy.

Errors of N.A. Tabular Place of Saturn.

		Tab.—Obs.	
		R.A.	Dec.
		^s	
July	24	+ '01	— 0''3
Aug.	18	— '06	'5
	19	'05	'5
	19	'06	'1
Sept.	9	'09	'6
	23	'08	'5
Oct.	2	'07	'4
	4	'04	'3
	12	'05	'7
	15	'05	'6
	30	— '02	— 0'8
Mean		— 8'051	— 0''48

Royal Observatory, Greenwich:
1908 January 8,

Observations of Occultations of Stars by the Moon, made at the Royal Observatory, Greenwich, in the Year 1907.

(Communicated by the Astronomer Royal.)

Day, 1907.	Phenomenon.	Telescope.	Power.	Moon's Limb.	Mean Solar Time of Observation.		Observer.
					h	m s	
January 26	Disapp. ν Geminorum	Old Altazimuth ...	100	Dark	6 39	40'12	A. C.
26	" "	Sheepshanks Equatorial	100	"	6 39	39'76	J. E.
February 23	" ζ Geminorum	Astrographic Equatorial	225	"	7 10	30'37	W. S.
23	" Piazz I VI. 311	" "	225	"	7 11	40'18	W. S.
23 (b) (c)	" "	Sheepshanks Equatorial	100	"	7 11	40'46	J. E.
March 20	" m Tauri	Old Altazimuth ...	100	"	9 23	21'42	S. E.
20 (a)	" "	Astrographic Equatorial	225	"	9 23	20'59	H.
20	" "	Sheepshanks Equatorial	100	"	9 23	21'23	R. C.
21	" χ^1 Orionis	Old Altazimuth ...	100	"	7 14	42'47	S. E.
21 (a)	" "	Great Equatorial ...	670	"	7 14	42'57	W. B.
21	" "	Sheepshanks Equatorial	100	"	7 14	42'08	J. E.
21	" "	Astrographic Equatorial	225	"	7 14	41'89	W. S.
21 (a)	" "	Merz Refractor ...	250	"	7 14	41'92	R. F.
21	" B. D. + 19°, 1146	Astrographic Equatorial	225	"	9 32	7'46	W. S.
21	" B. D. + 20°, 1202	Merz Refractor ...	250	"	10	6 16'43	C. D.
21	" B. D. + 20°, 1209	Astrographic Equatorial	225	"	10 37	8'75	W. S.
21	" B. D. + 20°, 1211	" "	225	"	10 46	22'22	W. S.
21	" χ^4 Orionis	Merz Refractor ...	250	"	12 28	49'33	R. F.

Day, 1907.	Phenomenon.	Telescope.	Power.	Moon's Limb.	Mean Solar Time of Observation.	Observer.
March						
21	Disapp. χ^4 Orionis	Sheepshanks Equatorial	100	Dark	h m s 12 28 51.23	J. E.
22	" B.D. + 20°, 1578	Astrographic Equatorial	225	"	9 25 16.99	W. S.
22	" B.D. + 20°, 1590	"	225	"	11 17 9.63	W. S.
23	" Piazzi VII. 44	"	225	"	7 16 13.93	W. S.
23	" B.D. + 20°, 1854	"	225	"	7 30 46.53	W. S.
23	" B.D. + 20°, 1866	"	225	"	8 39 26.01	W. S.
24	" B.D. + 19°, 2021	"	225	"	8 27 42.81	W. S.
24	" B.D. + 19°, 2024	"	225	"	8 57 22.65	W. S.
24	" B.D. + 18°, 1978	"	225	"	11 0 43.41	W. S.
28	" δ Virginis ...	"	225	"	12 12 27.99	A. L.
April						
19 (a)	" 56 Geminorum	"	225	"	9 23 17.04	H. F.
May	" α Virginis ...	Sheepshanks Equatorial	120	"	10 18 19.31	S. E.
June	" B.D. - 13°, 4111	Astrographic Equatorial	225	"	9 40 18.98	A. L.
22	" " "	Sheepshanks Equatorial	100	"	9 40 16.71	J. S.
24	" ξ Ophiuchi ...	Thompson Equatorial (Hodgson)	100	"	11 39 48.01	D. E.
24	" " "	Astrographic Equatorial	225	"	11 39 48.13	J. S.
24	" " "	Sheepshanks Equatorial	100	"	11 39 48.03	R. C.
September						
24	" μ Ceti ...	"	100	Bright	12 50 34.00	R. F.
24 (a)	Reapp. " "	Astrographic Equatorial	225	Dark	13 45 30.90	W.

Day, 1907.	Phenomenon.	Telescope.	Power.	Moon's Limb.	Mean Solar Time of Observation.	Observer.
September 26	Disapp. δ^2 Tauri	...	...	...	h m s	...
October 2	Reapp. 8 Leonis	...	...	...	11 57 3'23	S. E.
24	" ϵ^1 Tauri	...	...	...	17 11 8'27	H. F.
December 11	Disapp. Lalande 45490	...	...	...	10 18 8'18	W. S.
11	" "	...	...	...	6 6 18'79	A. C.
17	" δ^1 Tauri	...	...	...	6 6 18'54	A. L.
17	" "	...	...	...	12 52 16'14	S. E.
17	" "	...	...	...	12 52 16'20	A. C.
17	" δ^3 Tauri	...	...	...	12 52 16'40	R. C.
17	" "	...	...	...	14 8 18'08	S. E.

(c) Very faint, ill-defined.

(b) Observation doubtful.

The apertures of the telescopes used are as follow:—

Great Equatorial	...	...	...	Great Equatorial (Corbett Telescope)	...	6½ inches.
Merz Refractor	...	...	...	Thompson Equatorial (Hodgson Telescope)	...	6 "
Astrographic Equatorial (Guiding Telescope)	...	...	...	Old Altazimuth	...	4 "
Sheepshanks Equatorial	...	...	...			

The initials, S. E., H., A. C., C. D., D. E., W. B., H. F., W., J. S., J. E., R. C., R. F., A. L., are those of Mr. Eddington, Mr. Hollis, Mr. Crommelin, Mr. Davidson, Mr. Edney, Mr. Bowyer, Mr. Furner, Mr. Witchell, Mr. Storey, Mr. Stevens, Mr. J. Evans, Mr. Cullen, Mr. Fowler, and Mr. Lyne, respectively.

Royal Observatory, Greenwich:
1908 January 7.

Ephemeris of Flora near the time of Opposition in 1908. By
A. M. W. Downing, D.Sc., F.R.S.

This ephemeris is computed from Brünnow's *Tafeln der Flora*, in combination with the corrected continuation of certain of the tables published in *Monthly Notices*, vol. lxiv., No. 6.

Berlin Midnight, 1908.	Apparent		Log Distance from Earth.
	R.A.	Dec.	
	h m s		
May 1	15 56 44.46	- 12 5 29.7	0.19103
2	55 49.64	2 26.6	.18968
3	54 53.70	11 59 24.5	.18840
4	53 56.71	56 23.6	.18720
5	52 58.73	53 24.0	.18607
6	51 59.82	50 25.9	.18500
7	51 00.05	47 29.6	.18400
8	49 59.53	44 35.2	.18308
9	48 58.30	41 42.9	.18223
10	47 56.45	38 52.9	.18146
11	46 54.07	36 5.5	.18077
12	45 51.23	33 21.0	.18015
13	44 47.96	30 39.6	.17961
14	43 44.29	28 1.5	.17915
15	42 40.27	25 27.0	.17876
16	41 35.98	22 56.2	.17845
17	40 31.51	20 29.3	.17822
18	39 26.95	18 6.4	.17806
19	38 22.39	15 48.0	.17798
20	37 17.92	13 34.2	.17798
21	36 13.61	11 25.1	.17806
22	35 9.50	9 20.8	.17821
23	34 5.68	7 21.8	.17844
24	33 2.25	5 28.2	.17875
25	31 59.24	3 40.2	.17914
26	30 56.74	1 58.0	.17960
27	29 54.78	11 0 21.8	.18014
28	28 53.47	10 58 51.8	.18075
29	27 52.91	57 28.2	.18143
30	26 53.13	56 11.2	.18219
31	15 25 54.19	- 10 55 0.8	.18302

Berlin Midnight. 1908.		Apparent		Log Distance from Earth.
		R.A. h m s	Dec. ° ' "	
June	1	15 24 56.11	- 10 53 57.0	0.18392
	2	23 59.00	53 0.3	.18489
	3	23 2.97	52 10.9	.18592
	4	22 8.05	51 28.9	.18702
	5	21 14.27	50 54.2	.18819
	6	20 21.64	50 26.9	.18942
	7	19 30.25	50 7.1	.19071
	8	18 40.18	49 55.0	.19206
	9	17 51.47	49 50.7	.19347
	10	17 4.13	49 54.1	.19493
	11	16 18.17	50 5.2	.19645
	12	15 33.62	50 24.0	.19802
	13	14 50.56	50 50.6	.19964
	14	14 9.03	51 25.2	.20131
	15	15 13 29.07	- 10 52 7.7	0.20304

Magnitude at opposition, May 18 = 9.7.

